Demonstration of Newton's Method.

> $f := x \to x^6 - x - 1$

$$f := x \to x^6 - x - 1$$

> $fp := x \to 6 \cdot x^5 - 1$

$$fp := x \to 6\,x^5 - 1$$

> $F := x \to x - \frac{f(x)}{fp(x)}$

$$F := x \to x - \frac{f(x)}{fp(x)}$$

> $F(3.)$

$$2.502402196$$

> $F(3)$

$$\frac{3646}{1457}$$

> $x0 := 1.0$

$$x0 := 1.0$$

> $x1 := F(x0)$

$$x1 := 1.200000000$$

> $x2 := F(x1)$

$$x2 := 1.143575842$$

> $x3 := F(x2)$

$$x3 := 1.134909462$$

> $x4 := F(x3)$

$$x4 := 1.134724221$$

> $x5 := F(x4)$

$$x5 := 1.134724138$$

> $x6 := F(x5)$

$$x6 := 1.134724138$$

> $f(x6)$

$$-4.\ 10^{-9}$$

> $x7 := F(x6)$

$$x7 := 1.134724138$$

> $f(x7)$

$$-4.\ 10^{-9}$$

> $y0 := -1.0$

$$y0 := -1.0$$

> $y1 := F(y0)$

$$y1 := -.8571428571$$

> $y2 := F(y1)$

$$y2 := -.7899518503$$

> $y3 := F(y2)$

$$y3 := -.7783727110$$

> $y4 := F(y3)$

$$y4 := -.7780897613$$

> $y5 := F(y4)$

$$y5 := -.7780895986$$

> $y6 := F(y5)$

$$y6 := -.7780895987$$

> $f(y6)$

$$0$$
$$.$$

>

Next example: #6, p. 329

> $f := x \rightarrow \arctan(x) - 1 + 2 \cdot x$

$$f := x \rightarrow \arctan(x) - 1 + 2\,x$$

> $fp := x \rightarrow \frac{1}{(1 + x^2)} + 2$

$$fp := x \rightarrow \frac{1}{1 + x^2} + 2$$

> $F := x \rightarrow x - \frac{f(x)}{fp(x)}$

$$F := x \rightarrow x - \frac{f(x)}{fp(x)}$$

> $z0 := .25$

$$z0 := 0.25$$

> $z1 := F(z0)$

$$z1 := 0.3367072545$$

> $z2 := F(z1)$

$$z2 := 0.3373288487$$

> $z3 := F(z2)$

$$z3 := 0.3373288849$$

>

Correct answer is z*=.337329 to 6 dec. places

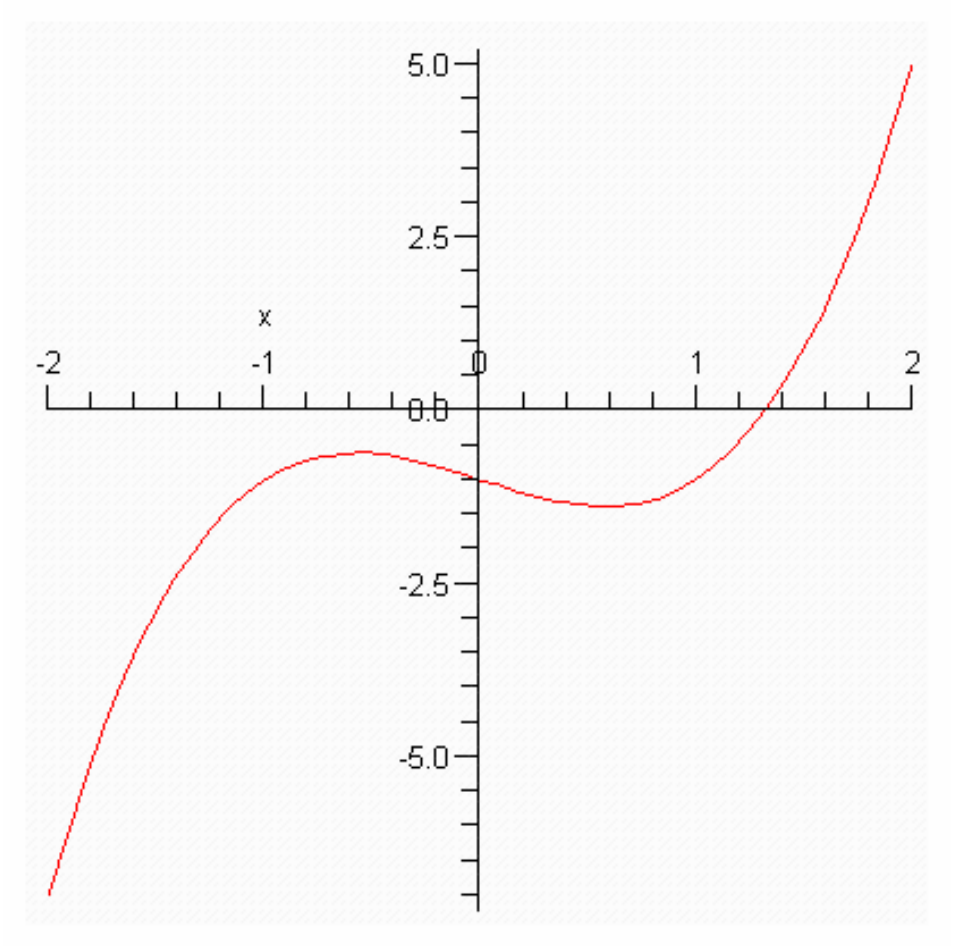
>

Another example.

> $f := x \rightarrow x^3 - x - 1$

$$f := x \rightarrow x^3 - x - 1$$

> $plot(f(x), x = -2..2)$



> $F := x \to x - \frac{f(x)}{3 \cdot x^2 - 1}$

$$F := x \to x - \frac{f(x)}{3\,x^2 - 1}$$

> $x0 := 1.$

$$x0 := 1.$$

> $x1 := F(x0)$

$$x1 := 1.500000000$$

> $x2 := F(x1)$

$$x2 := 1.347826087$$

> $x3 := F(x2)$

$$x3 := 1.325200399$$

> $x4 := F(x3)$

$$x4 := 1.324718174$$

> $x5 := F(x4)$

$$x5 := 1.324717957$$

> $y0 := .6$

$$y0 := 0.6$$

> $y1 := F(y0)$

$$y1 := 17.90000000$$

> $y2 := F(y1)$

$$y2 := 11.94680233$$

> $y3 := F(y2)$

$$y3 := 7.985520351$$

> $y4 := F(y3)$

$$y4 := 5.356909314$$

> $y5 := F(y4)$

$$y5 := 3.624996032$$

> $y6 := F(y5)$

$$y6 := 2.505589189$$

> $y7 := F(y6)$

$$y7 := 1.820129422$$

>

Looks like we are converging.

Rewrite previous problem as x^2-1-(1/x)=0.

> $g := x \to x^2 - 1 - \frac{1}{x}$

$$g := x \to x^2 - 1 - \frac{1}{x}$$

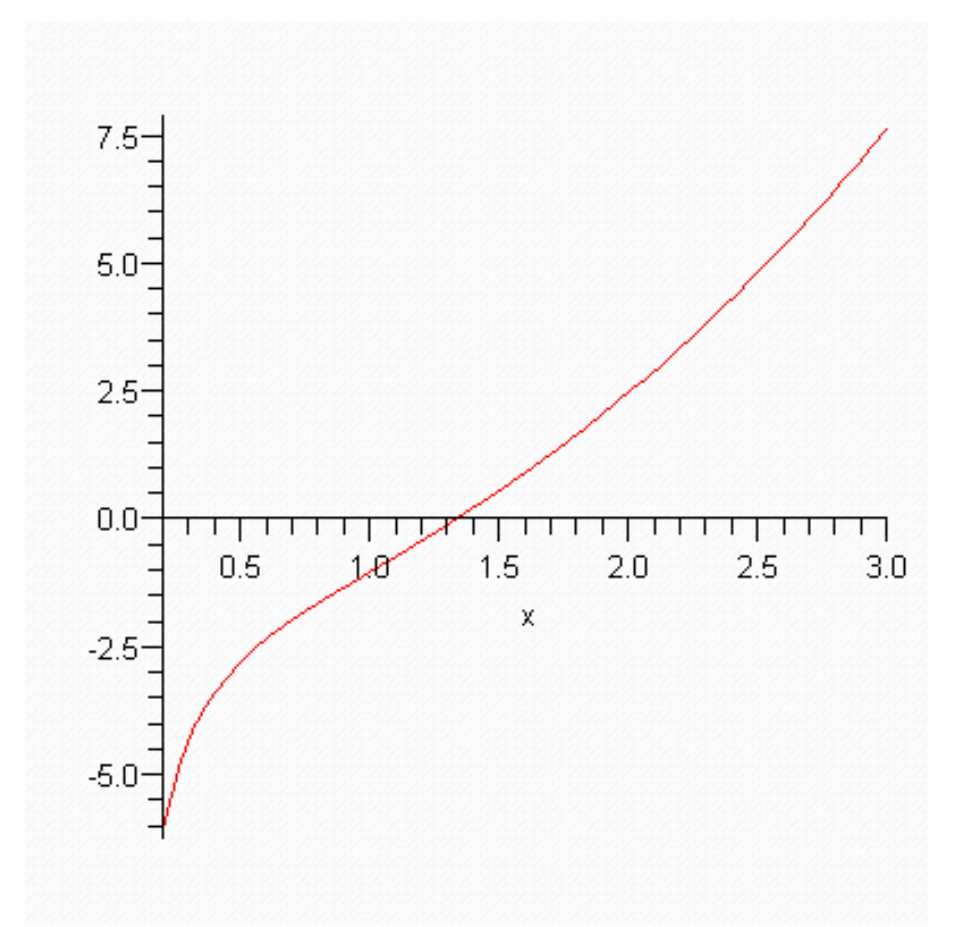
> $gp := x \to D(g)(x)$

$$gp := x \to (\mathrm{D}(g))(x)$$

> $G := x \to x - \frac{g(x)}{gp(x)}$

$$G := x \to x - \frac{g(x)}{gp(x)}$$

> $plot(g(x), x = .2..3)$



> $z0 := .5$

$$z0 := 0.5$$

> $z1 := G(z0)$

$$z1 := 1.050000000$$

> $z2 := G(z1)$

$$z2 := 1.332631400$$

> $z3 := G(z2)$

$$z3 := 1.324729110$$

> $z4 := G(z3)$

$$z4 := 1.324717957$$

>

We are in the zone.