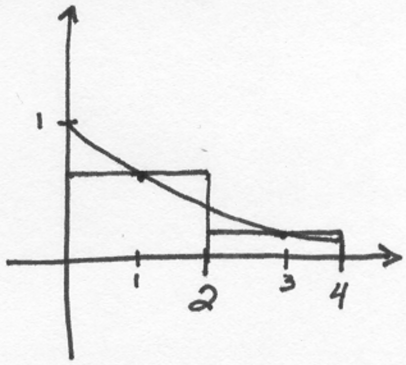
MATH 113 – QUIZ 12 – 23 APRIL 2007

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Approximate the area under the graph of $f(x) = \dfrac{1}{1+x}$ over the interval $[0, 4]$ using the Midpoint Rule with $n = 2$ and $n = 4$ (that is by dividing the interval $[0, 4]$ into n equal subintervals and using the midpoint of each subinterval in the approximation).

$f(x) = \dfrac{1}{1+x}$

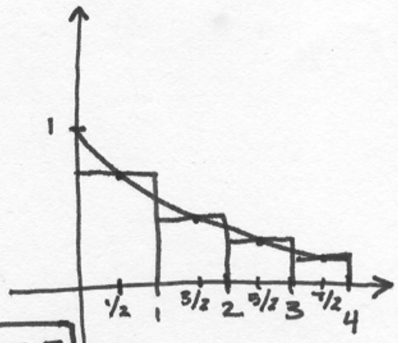


$n=2$: $\Delta x = \dfrac{4-0}{2} = 2$

$A = 2(f(1)) + 2(f(3))$

$= 2\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) = 1 + 1/2 = \boxed{3/2}$

$n=4$: $\Delta x = \dfrac{4-0}{4} = 1$



$A = 1(f(1/2)) + 1(f(3/2)) + 1(f(5/2)) + 1(f(7/2))$

$= \dfrac{1}{1+1/2} + \dfrac{1}{1+3/2} + \dfrac{1}{1+5/2} + \dfrac{1}{1+7/2}$

$= \dfrac{1}{3/2} + \dfrac{1}{5/2} + \dfrac{1}{7/2} + \dfrac{1}{9/2} = 2/3 + 2/5 + 2/7 + 2/9 = \boxed{1.575}$

2. (4 pts.) Given that $\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$ and $\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$, evaluate the sum $\sum_{k=1}^{10} k(3k+5)$.

$\sum_{K=1}^{10} K(3K+5) = \sum_{K=1}^{10} 3K^2 + 5K = \sum_{K=1}^{10} 3K^2 + \sum_{K=1}^{10} 5K$

$= 3\sum_{K=1}^{10} K^2 + 5\sum_{K=1}^{10} K$

$= 3\left(\dfrac{n(n+1)(2n+1)}{6}\right) + 5\left(\dfrac{n(n+1)}{2}\right) = 3\left(\dfrac{10(11)(21)}{6}\right) + 5\left(\dfrac{10(11)}{2}\right)$

$= 3\left(\dfrac{2310}{6}\right) + 5\left(\dfrac{110}{2}\right) = 1155 + 275 = \boxed{1430}$