Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. Consider the function $f(x) = 2x^3 - 6x^2 - 3$.

(a) (2 pts.) Find the first two derivatives of $f(x)$.

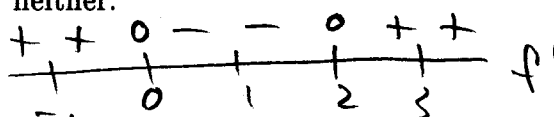$$f'(x) = 6x^2 - 12x$$
$$f''(x) = 12x - 12$$

(b) (3 pts.) Find all critical points of $f(x)$, and identify each point as the location of a local maximum, local minimum, or neither.

$$6x^2 - 12x = 0$$
$$6x(x-2) = 0$$

$$\boxed{\begin{array}{l} x = 0 \quad x = 2 \\ \text{crit. points} \end{array}}$$

$$f'(-1) > 0 \quad f'(1) < 0 \quad f'(3) > 0$$

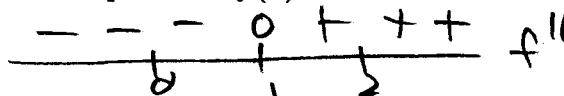$$\boxed{\begin{array}{l} \text{local max at } x = 0 \\ \text{local min at } x = 2 \end{array}}$$

(c) (2 pts.) Find the location of all inflection points of $f(x)$.

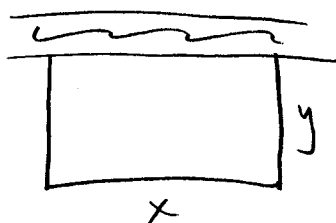$$12x - 12 = 0$$
$$12(x-1) = 0$$
$$x = 1$$

$$f''(0) < 0 \quad f''(2) > 0$$

$$\boxed{\text{inflection point at } x = 1}$$

2. (3 pts.) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a fence. If you have 800 meters of fence, what is the largest area that can be enclosed?

$$x + 2y = 800$$
$$y = \tfrac{1}{2}(800 - x)$$
$$= 400 - \tfrac{1}{2}x$$

$$A = xy$$
$$= x(400 - \tfrac{1}{2}x)$$
$$= -\tfrac{1}{2}x^2 + 400x$$

$$A' = -x + 400$$

$$-x + 400 = 0$$
$$\therefore x = 400$$

$$y = 400 - \tfrac{1}{2}(400)$$
$$= 200$$

$$\therefore \text{largest area}$$
$$= (400)(200) = 80000$$
$$\text{sq meters}$$