MATH 113 – QUIZ 7 – 19 MARCH 2007

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (2 pts. each) Find the derivative of each of the following functions.

(a) $g(t) = \ln(t^2 + 3t)$

$$g'(t) = \frac{1}{t^2+3t} \cdot \frac{d}{dt}(t^2+3t) = \frac{2t+3}{t^2+3t} \;//$$

(b) $q(r) = \sin^{-1}(r^2)$

$$q'(r) = \frac{1}{\sqrt{1-(r^2)^2}} \cdot \frac{d}{dr}(r^2) = \frac{2r}{\sqrt{1-r^4}} \;//$$

2. (3 pts.) Use logarithmic differentiation to find the derivative of $y = x\sqrt{x^2+1}$.

$$\ln(y) = \ln\left(x \cdot (x^2+1)^{1/2}\right) = \ln x + \tfrac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+1} = \frac{1}{x} + \frac{x}{x^2+1}$$

$$\therefore \frac{dy}{dx} = x(x^2+1)^{1/2}\left(\frac{1}{x} + \frac{x}{x^2+1}\right) \;//$$

3. (3 pts.) Find the equation of the tangent line to the curve defined by the equation $2y^{1/2} = x - y$ at the point $(3, 1)$.

$$\frac{d}{dx}(2y^{1/2}) = \frac{d}{dx}(x-y)$$

$$y^{-1/2}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx}(1 + y^{-1/2}) = 1$$

$$\frac{dy}{dx} = \frac{1}{1+y^{-1/2}} \;//$$

$$\left.\frac{dy}{dx}\right|_{(3,1)} = \frac{1}{1+(1)^{-1/2}} = \frac{1}{2}$$

Tangent line: slope $= \frac{1}{2}$, point $= (3,1)$

$$y = 1 + \tfrac{1}{2}(x-3) \;//$$

$$= \tfrac{1}{2}x - \tfrac{1}{2} \;//$$