>

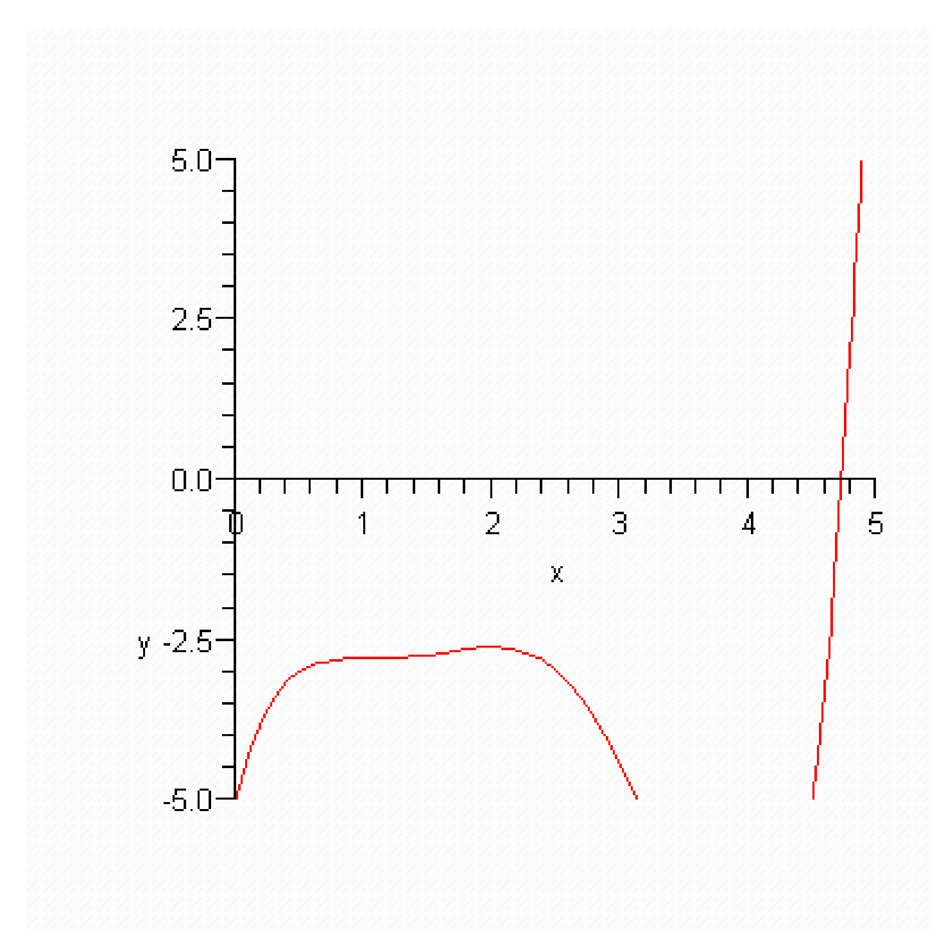
MAPLE Assignment #4 -- Solutions

#1(a).

> $f := x \rightarrow \left(\frac{1}{5}\right) \cdot x^5 - 2 \cdot x^4 + 7 \cdot x^3 - 11 \cdot x^2 + 8 \cdot x - 5$

$$f := x \rightarrow \frac{1}{5} x^5 - 2 x^4 + 7 x^3 - 11 x^2 + 8 x - 5$$

> $plot(f(x), x = 0..5, y = -5..5)$



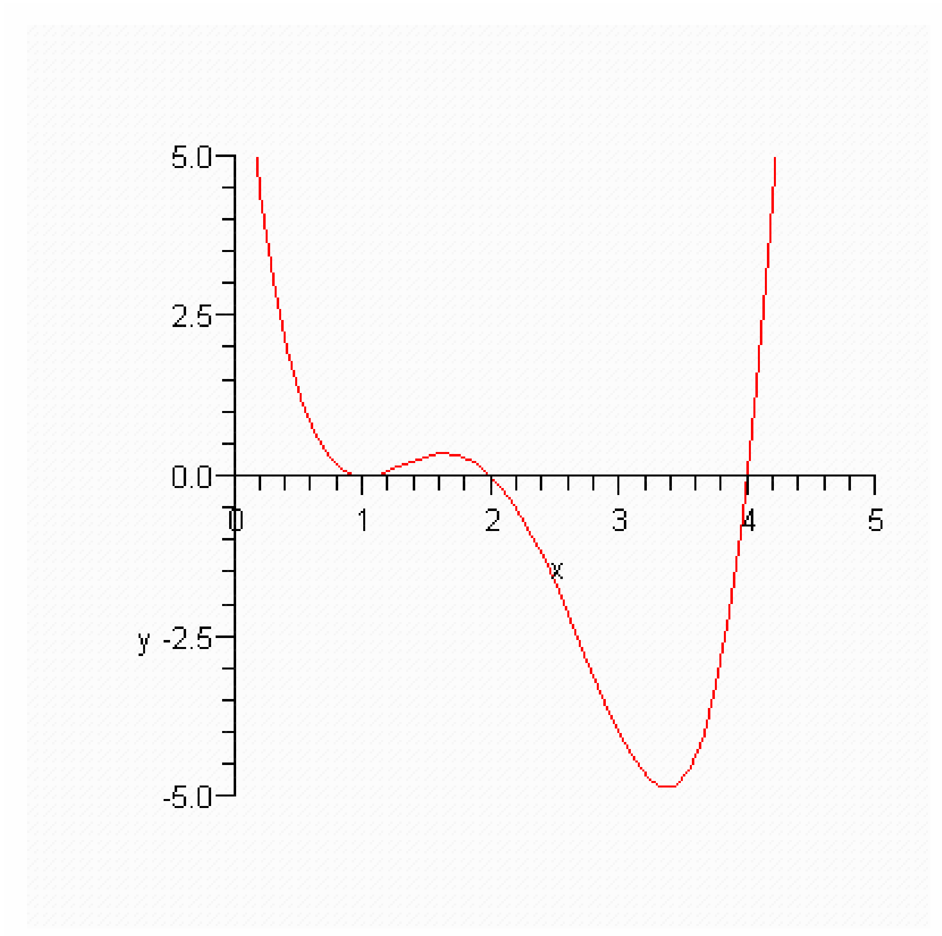
> $fp := x \rightarrow D(f)(x)$

$$fp := x \rightarrow (\mathrm{D}(f))(x)$$

> $eval(fp(x))$

$$x^4 - 8x^3 + 21x^2 - 22x + 8$$

> $plot(fp(x), x = 0..5, y = -5..5)$



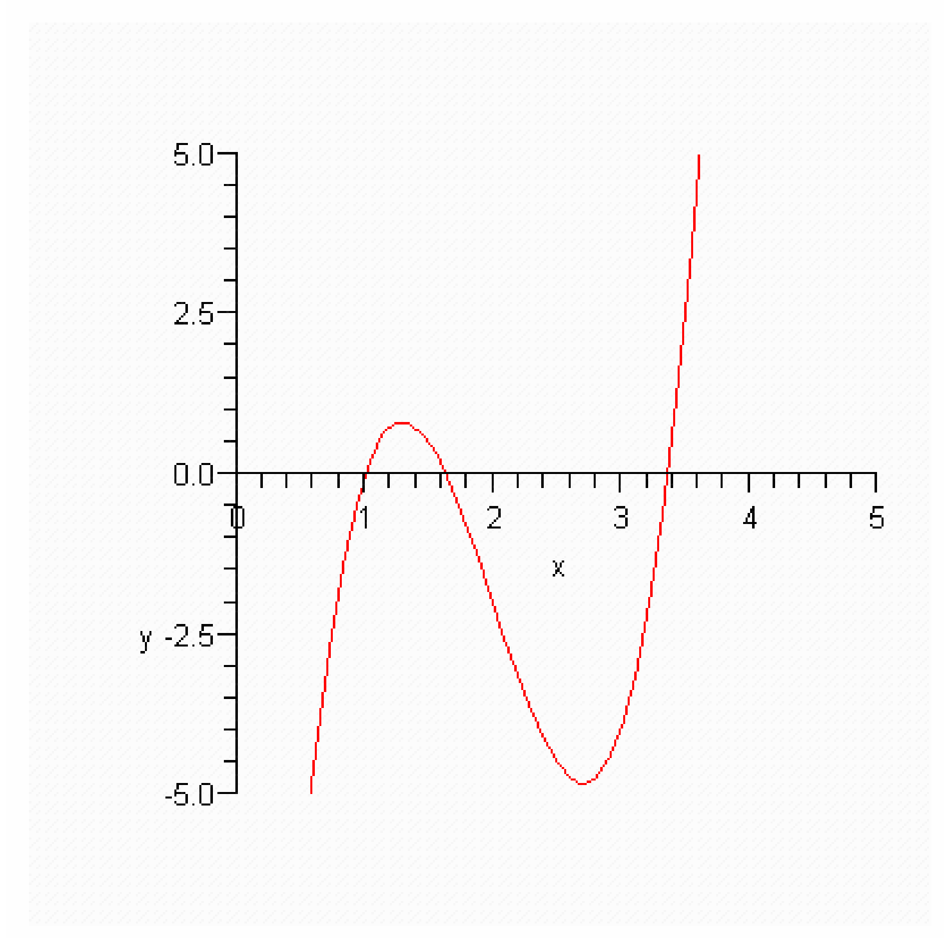
> $fpp := x \rightarrow D(fp)(x)$

$$fpp := x \rightarrow (D(fp))(x)$$

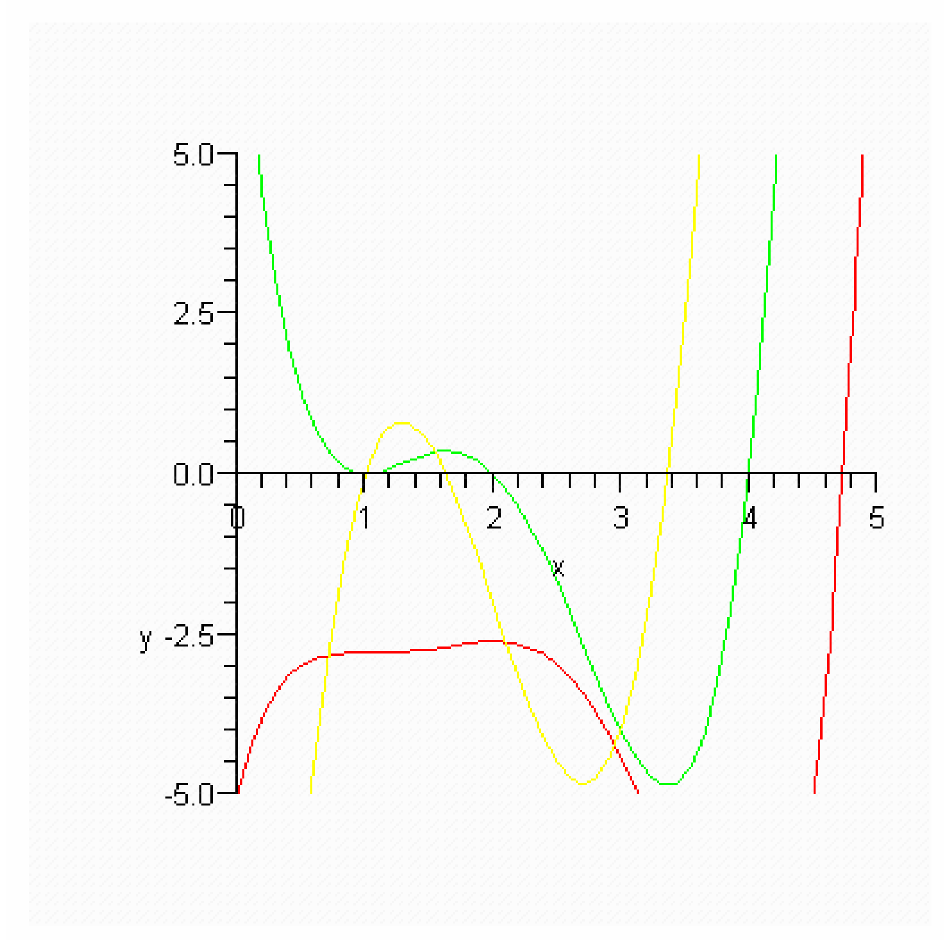
> $eval(fpp(x))$

$$4x^3 - 24x^2 + 42x - 22$$

> $plot(fpp(x), x = 0..5, y = -5..5)$



> $plot([f(x), fp(x), fpp(x)], x = 0..5, y = -5..5)$



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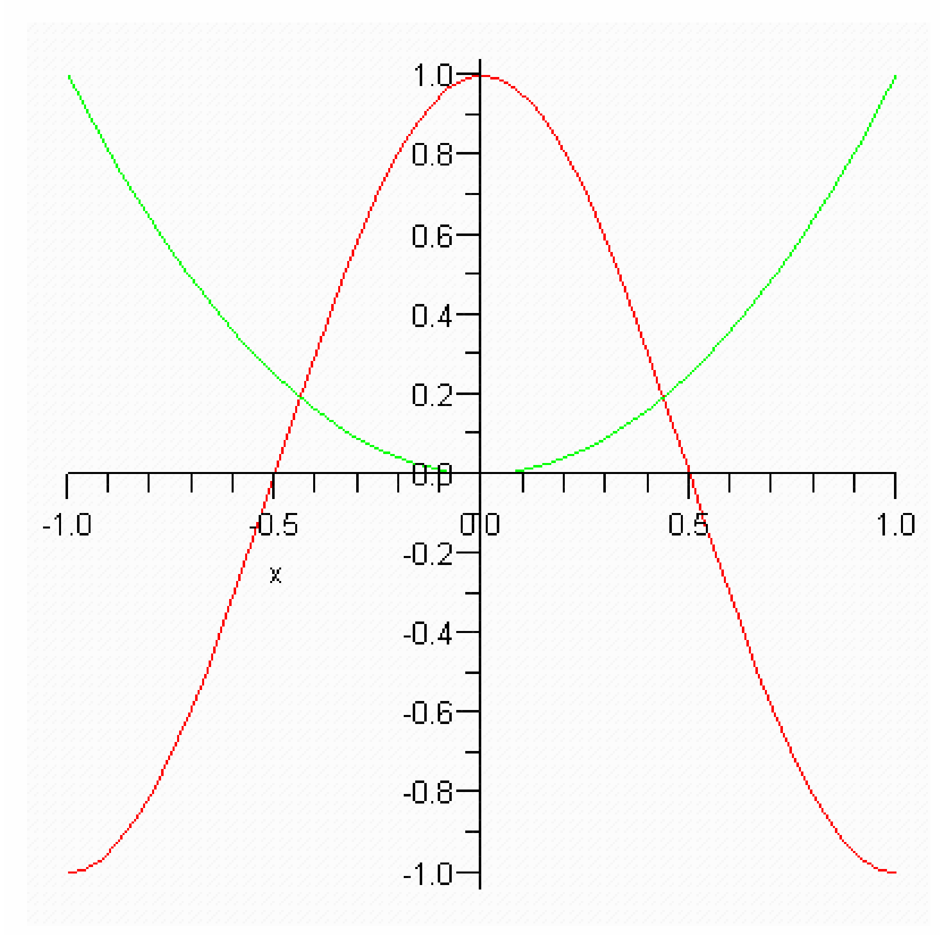
#1(b).

f(x) is increasing on (0,2) U (4,5)
f(x) is decreasing on (2,4)

f(x) is concave up on (1,1.6) U (3.4,5)
f(x) is concave down on (0,1) U (1.6,3.4)

#2(a).

> $plot([cos(Pi\cdot x), x^2], x = -1..1)$



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Clearly there is only one positive solution located somewhere between 0 and 0.5.

> $fsolve(cos(Pi \cdot x) = x^2, x)$

0
.4384307795

>

#2(b).

> $f := x \rightarrow cos(Pi \cdot x) - x^2$

$$f := x \rightarrow \cos(\pi x) - x^2$$

> $fp := x \rightarrow D(f)(x)$

$$fp := x \rightarrow (\mathrm{D}(f))(x)$$

> $eval(fp(x))$

$$-\sin(\pi x)\,\pi - 2x$$

>

> $F := x \rightarrow x - \frac{f(x)}{fp(x)}$

$$F := x \rightarrow x - \frac{f(x)}{fp(x)}$$

> $eval(F(x))$

$$x - \frac{\cos(\pi x) - x^2}{-\sin(\pi x)\,\pi - 2x}$$

>

#2(c).

Starting at x0=0.5

> $x0 := 0.5$

$$x0 := 0.5$$

> $x1 := evalf(F(x0))$

$$x1 := 0.4396367482$$

> $x2 := evalf(F(x1))$

$$x2 := 0.4384314896$$

> $x3 := evalf(F(x2))$

$$x3 := 0.4384307794$$

> $x4 := evalf(F(x3))$

$$x4 := 0.4384307794$$

> $x5 := evalf(F(x4))$

$$x5 := 0.4384307794$$

>

Starting at x0=0.01

> $y0 := 0.01$

$$y0 := 0.01$$

> $y1 := evalf(F(y0))$

$$y1 := 8.431032696$$

> $y2 := evalf(F(y1))$

$$y2 := 4.875257208$$

> $y3 := evalf(F(y2))$

$$y3 := 2.620333927$$

> $y4 := evalf(F(y3))$

$$y4 := 1.733708115$$

> $y5 := evalf(F(y4))$

$$y5 := -.323786161$$

> $y6 := evalf(F(y5))$

$$y6 := -.4505681378$$

> $y7 := evalf(F(y6))$

$$y7 := -.4384979147$$

> $y8 := evalf(F(y7))$

$$y8 := -.4384307815$$

> $y9 := evalf(F(y8))$

$$y9 := -.4384307795$$

> $y10 := evalf(F(y9))$

$$y10 := -.4384307795$$

>

Newton's method converges for both initial values. For x0=0.5 it converges to the positive root and for y0=0.01 it converges to the negative root (the one we did not want).

The initial value y0=0.01 took much longer to converge to the root because the starting value was very close to a point where fp(x) was zero. Note that fp(x) = -pi*sin(pi*x)-2x is zero when x=0. The first iterate y1 was very large because of this. The method eventually converged but to the wrong root.