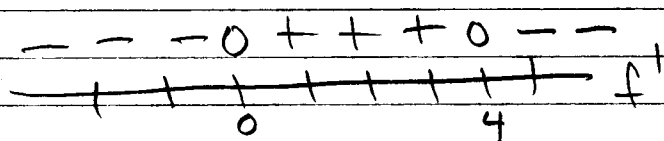


MATH 113 - EXAM 3 - SOLUTIONS

1. $V = \frac{4}{3}\pi r^3$ $dV = 4\pi r^2 dr$

$\Delta V \approx dV = 4\pi(3)^2(-.2) = -7.2\pi \approx -22.6 \text{ m}^3 //$

2. (a) $f'(x) = 20x^3 - 5x^4$ critical points:



$20x^3 - 5x^4 = 0$

$5x^3(4-x) = 0$

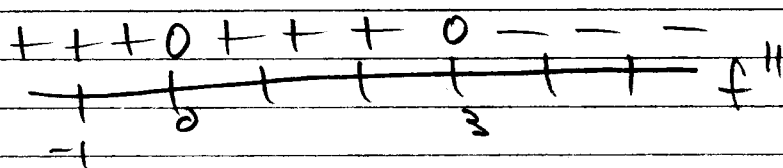
$x=0$ $x=4$

$f(x)$ incr on $(0, 4)$

$f(x)$ decr on $(-\infty, 0) \cup (4, \infty) //$

Local min at $x=0 //$ local max at $x=4 //$

(b) $f''(x) = 60x^2 - 20x^3$ Possible inflection pts:



$60x^2 - 20x^3 = 0$

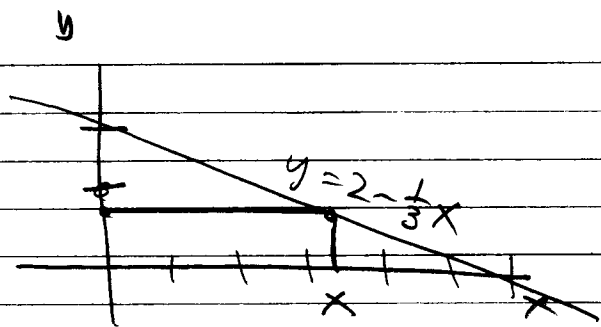
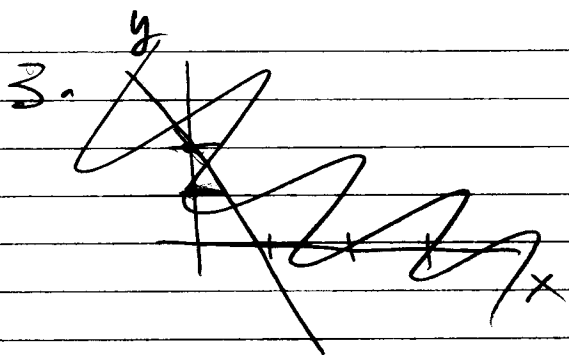
$20x^2(3-x) = 0$

$x=0$ $x=3$

$f(x)$ concave up on $(-\infty, 3)$

$f(x)$ concave down on $(3, \infty) //$

inflection point at $x=3 //$



$$A = x(2 - \frac{1}{3}x) = -\frac{1}{3}x^2 + 2x.$$

$$A' = -\frac{2}{3}x + 2 \quad -\frac{2}{3}x + 2 = 0$$

$$\frac{2}{3}x = 2$$

$$x = 3 \quad y = 2 - \frac{1}{3} \cdot 3 = 1 //$$

$$4. (a) \lim_{t \rightarrow 1} \frac{t^3 - 1}{4t^3 - t - 3} \stackrel{L'H}{=} \lim_{t \rightarrow 1} \frac{3t^2}{12t^2 - 1} = \frac{3}{11} //$$

$$(b) \lim_{x \rightarrow 0^+} \sqrt{x} \ln(x) = \lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^{-1/2}} \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}}$$

$$= -2 \lim_{x \rightarrow 0^+} \frac{x^{3/2}}{x} = -2 \lim_{x \rightarrow 0^+} x^{1/2} = 0 //$$

$$5. (a) \quad x^3 - 2x - 2 = 0$$

$$f(x) = x^3 - 2x - 2$$

$$f'(x) = 3x^2 - 2$$

$$x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 2}{3x_n^2 - 2} = \frac{2x_n^3 + 2}{3x_n^2 - 2} //$$

$$(b) \quad x_0 = 2$$

$$x_1 = \frac{2x_0^3 + 2}{3x_0^2 - 2} = \frac{2(2)^3 + 2}{3(2)^2 - 2} = \frac{18}{10} = \frac{9}{5} = 1.8 //$$

$$x_2 = \frac{2\left(\frac{9}{5}\right)^3 + 2}{3\left(\frac{9}{5}\right)^2 - 2} = \frac{2 \cdot \frac{729}{125} + 2}{3 \cdot \frac{81}{25} - 2} \cdot \frac{125}{125}$$

$$= \frac{2 \cdot 729 + 2 \cdot 125}{15 \cdot 81 - 2 \cdot 125} = \frac{1708}{965} = 1.76995 //$$

$$6. (a) \quad \int (x^4 + x^{-1/2}) dx = \frac{1}{5}x^5 + 2x^{1/2} + C //$$

$$(b) \quad \int 3 \cos(3\theta) d\theta = \sin(3\theta) + C //$$

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