

MATH 113 – 18 APRIL 2007 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given.

1. (12 pts.) The volume, V (in cubic meters), of a sphere of radius r meters is given by $V = (4/3)\pi r^3$. Use *differentials* to estimate the change ΔV in the volume of a sphere when the radius decreases from 3 meters to 2.8 meters.

2. (12 pts. each) Let $f(x) = 5x^4 - x^5$.

(a) Find the intervals on which $f(x)$ is increasing and decreasing, and the location of all local maxima and minima for $f(x)$.

(b) Find the intervals on which $f(x)$ is concave up and concave down, and the location of all inflection points of $f(x)$.

3. (12 pts.) Find the dimensions of the rectangle of largest area that can be inscribed in the region in the first quadrant bounded by the x -axis, the y -axis and the line $y = 2 - (1/3)x$.

4. (12 pts. each) Find the following limits using L'Hopital's rule.

(a) $\lim_{t \rightarrow 1} \frac{t^3 - 1}{4t^3 - t - 3}$

(b) $\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$

5. (a) (8 pts.) Set up the Newton's method iteration scheme to find the solution to the equation $x^3 - 2x - 2 = 0$.

(b) (4 pts.) Using the scheme you found in part (a), find the first two Newton iterates (that is, x_1 and x_2) when $x_0 = 2$. Write your answers correct to five decimal places.

6. (8 pts. each) Compute the following indefinite integrals.

(a) $\int (x^4 + x^{-1/2}) dx$.

(b) $\int 3 \cos(3\theta) d\theta$.