MATH 113 - EXAM 2 - SOLUTIONS

1. (a) $f(x) = x^3 - 3(x^2+4)$

$= x^3 - 3x^2 - 12$

$f'(x) = 3x^2 - 6x$ //

(b) $g(t) = (e^{-t}+3)\tan(t)$

$g'(t) = (e^{-t}+3)\frac{d}{dt}\tan(t) + \frac{d}{dt}(e^{-t}+3)\tan(t)$

$= (e^{-t}+3)\sec^2(t) - e^{-t}\tan(t)$ //

(c) $r(\theta) = \frac{1+\sin(\theta)}{1-\cos(\theta)}$

$r'(\theta) = \frac{(1-\cos(\theta))(\cos(\theta)) - (1+\sin(\theta))(\sin(\theta))}{(1-\cos(\theta))^2}$

$= \frac{\cos(\theta) - \cos^2(\theta) - \sin(\theta) - \sin^2(\theta)}{(1-\cos(\theta))^2}$

$= \frac{\cos(\theta) - \sin(\theta) - 1}{(1-\cos(\theta))^2}$ //

(d) $f(x) = (5x^2 + \ln(2x))^{3/2}$

$f'(x) = \frac{3}{2}(5x^2+\ln(2x))^{1/2}(10x + \frac{2}{2x})$

$= \frac{3}{2}(10x + \frac{1}{x})(5x^2+\ln(2x))^{1/2}$ //

(e) $h(x) = \sin^{-1}(x^2-1)$

$h'(x) = \frac{1}{\sqrt{1-(x^2-1)^2}} \cdot 2x$

$= \frac{2x}{\sqrt{1-(x^4-2x^2+1)}}$

$= \frac{2x}{\sqrt{2x^2-x^4}}$ //

(f) $f(x) = (x+2)^x = e^{x\ln(x+2)}$

$f'(x) = e^{x\ln(x+2)}\left(x\cdot\frac{1}{x+2} + \ln(x+2)\cdot 1\right)$

$= (x+2)^x\left(\frac{x}{x+2} + \ln(x+2)\right)$

2. (a) $s(t) = \frac{1}{4}t^4 - t^3 + t^2$

$v(t) = s'(t) = t^3 - 3t^2 + 2t$ //

$a(t) = v'(t) = s''(t) = 3t^2 - 6t + 2$ //

(b) Solve $v(t) = 0$:

$t^3 - 3t^2 + 2t = 0$

$t(t^2 - 3t + 2) = 0$

$t(t-1)(t-2) = 0$

$\therefore$ Body at rest when $t = 0, 1, 2$ //.

(c) $v(t) > 0$ when t in $(0,1) \cup (2,3)$

so body moves right

$v(t) < 0$ when t in $(1,2)$ so body moves left.

3 (a) $\frac{d}{dx}(x^3 + 4xy - 3y^{4/3}) = \frac{d}{dx}(2x+9)$

$3x^2 + 4x\frac{dy}{dx} + 4y - 4y^{1/3}\frac{dy}{dx} = 2$

$\frac{dy}{dx}(4x - 4y^{1/3}) = 2 - 3x^2 - 4y$

$\frac{dy}{dx} = \frac{2 - 3x^2 - 4y}{4x - 4y^{1/3}}$ //

(b) $\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{2 - 12 - 4}{8 - 4} = \frac{-7}{2}$

slope $\searrow$

point: $(2,1)$

$\therefore$ tangent line: $y = 1 - \frac{7}{2}(x-2)$ //

$= -\frac{7}{2}x + 8$ //

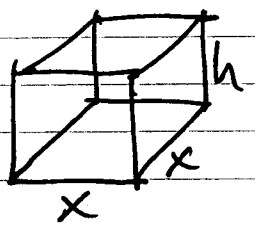
4. $y = \dfrac{(t+1)^3 \sin(t)}{(t^2+1)^{1/2}}$

$$\ln(y) = \ln\left(\frac{(t+1)^3 \sin t}{(t^2+1)^{1/2}}\right) = 3\ln(t+1) + \ln(\sin(t)) + \frac{1}{2}\ln(t^2+1)$$

$$\frac{1}{y}\frac{dy}{dt} = \frac{3}{t+1} + \frac{\cos(t)}{\sin(t)} + \frac{1}{2}\cdot\frac{2t}{t^2+1} = \frac{3}{t+1} + \cot(t) + \frac{t}{t^2+1}$$

$$\frac{dy}{dt} = \frac{(t+1)^3 \sin(t)}{(t^2+1)^{1/2}}\left(\frac{3}{t+1} + \cot(t) + \frac{t}{t^2+1}\right) //$$

5.



$V = x^2 h$

$$\frac{dV}{dt} = x^2\frac{dh}{dt} + 2x\frac{dx}{dt}\cdot h$$

$\dfrac{dV}{dt} = 1200$ $\dfrac{dx}{dt} = -20$ $x = 10$ $h = 15$

$$\therefore \quad 1200 = (10)^2\frac{dh}{dt} + 2(10)(-20)(15)$$

$$1200 = 100\frac{dh}{dt} - 6000$$

$$7200 = 100\, dh/dt \qquad \therefore \frac{dh}{dt} = 72\ \frac{cm}{min} //$$