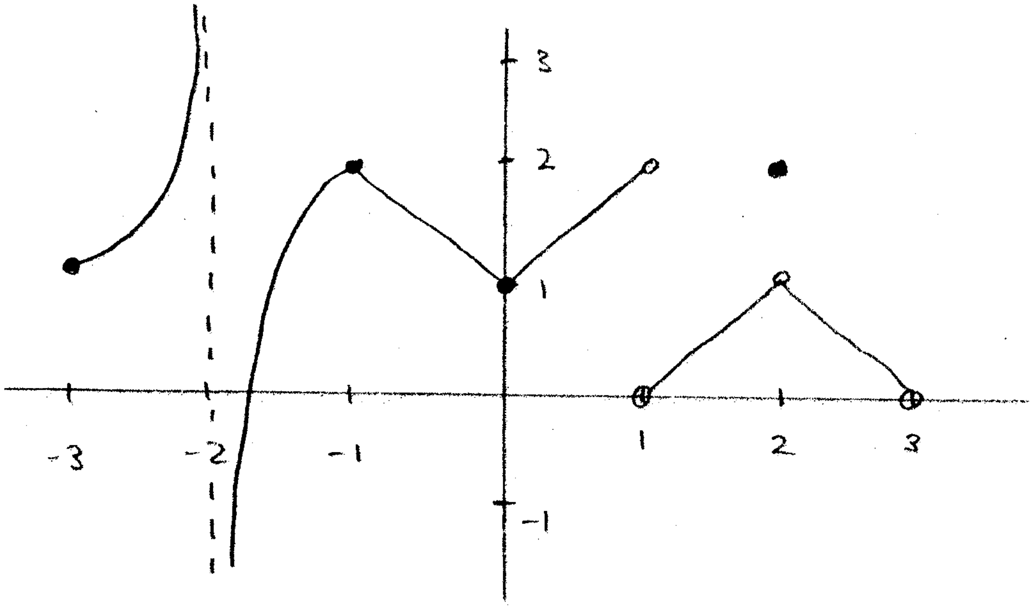
MATH 113 – 27 SEPTEMBER 2005 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given.

1. (3 pt. each) Consider the function $f(x)$ whose graph is sketched below. Determine whether each of the following statements is true or false (CIRCLE ONE).



$\lim_{x\to 2} f(x) = 2.$	T F	$\lim_{x\to 2} f(x) = f(2).$	T F
$f(x)$ is continuous at $x = 2.$	T F	$\lim_{x\to 1^-} f(x)$ exists.	T F
$\lim_{x\to 1^+} f(x)$ exists.	T F	$\lim_{x\to 1} f(x)$ exists.	T F
$f(x)$ is continuous at $x = 0.$	T F	$f(x)$ is differentiable at $x = 0.$	T F
$\lim_{x\to 3^-} f(x) = 0.$	T F	$\lim_{x\to -3^+} f(x) = 1.$	T F
$\lim_{x\to -2^+} f(x) = +\infty.$	T F	$\lim_{x\to -2^-} f(x) = +\infty.$	T F

2. (8 pts. each) Evaluate each of the following limits.

(a) $\displaystyle\lim_{y\to 0} \frac{y}{\sin(3y)}$

(b) $\displaystyle\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - 1}$

3. (8 pts. each) Let $f(x) = \dfrac{x^2 - 3x}{x^2 - 4}$. Evaluate each of the following limits. If the limit is infinite you must determine whether or not it is $+\infty$ or $-\infty$.

(a) Find $\lim\limits_{x \to -\infty} f(x)$

(b) Find $\lim\limits_{x \to 2^-} f(x)$

(c) Find $\lim\limits_{x \to 0} f(x)$

4. (12 pts.) Find the slope of the tangent line to the graph of the function $f(x) = \dfrac{x}{x-2}$ at the point $(3, 3)$ by calculating the limit of the difference quotient. Then find an equation for the line tangent to the graph there.

5. (12 pts.) Find the derivative $f'(x)$ of the function $f(x) = \sqrt{1+x}$ by using the definition of the derivative as the limit of the difference quotient.

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2. (a) $$\lim_{y\to 0} \frac{y}{\sin(3y)} = \lim_{y\to 0} \frac{1}{3}\cdot\frac{3y}{\sin(3y)}$$

$$= \frac{1}{3}\cdot \lim_{y\to 0}\frac{3y}{\sin(3y)} = \frac{1}{3} \,/\!/$$

(b) $$\lim_{x\to 1}\frac{x^2+x-2}{x^2-1} = \lim_{x\to 1}\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$= \lim_{x\to 1}\frac{x+2}{x+1} = \frac{3}{2} \,/\!/$$

3. (a) $$\lim_{x\to -\infty}\frac{x^2-3x}{x^2-4} = \lim_{x\to -\infty}\frac{x^2}{x^2} = 1 \,/\!/$$

<u>or</u> $$= \lim_{x\to -\infty}\frac{(x^2-3x)\cdot\frac{1}{x^2}}{(x^2-4)\cdot\frac{1}{x^2}} = \lim_{x\to -\infty}\frac{1-\frac{3}{x}}{1-\frac{4}{x^2}}$$

$$= 1 \,/\!/$$

(b) $\lim_{x \to 2^-} \frac{x^2-3x}{x^2-4} = +\infty$ //

If $x<2$ but near 2, $x^2-4<0$ and $x^2-3x<0$. Hence $\frac{x^2-3x}{x^2-4} > 0$.

(c) $\lim_{x \to 0} \frac{x^2-3x}{x^2-4} = 0$ // (substitution).

4. Slope: $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h \to 0} \frac{\frac{3+h}{3+h-2} - 3}{h}$

$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{3+h-3(1+h)}{1+h} = \lim_{h \to 0} \frac{\cancel{3}+h-\cancel{3}-3h}{h(1+h)}$

$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)} = -2$

Tangent line: $y = 3-2(x-3)$

$= -2x+9$ //

5.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{1+x+h}-\sqrt{1+x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+x+h}-\sqrt{1+x}}{h} \cdot \frac{\sqrt{1+x+h}+\sqrt{1+x}}{\sqrt{1+x+h}+\sqrt{1+x}}$$

$$= \lim_{h\to 0} \frac{\cancel{1+x}+h-\cancel{(1+x)}}{h(\sqrt{1+x+h}+\sqrt{1+x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{1+x+h}+\sqrt{1+x})}$$

$$= \frac{1}{2\sqrt{1+x}} \quad //$$