MATH 113 – 27 SEPTEMBER 2005 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given.

1. (3 pt. each) Consider the function $f(x)$ whose graph is sketched below. Determine whether each of the following statements is true or false (CIRCLE ONE).

$\lim_{x\to 2} f(x) = 2.$	**T**	**F**	$\lim_{x\to 2} f(x) = f(2).$	**T**	**F**
$f(x)$ is continuous at $x = 2.$	**T**	**F**	$\lim_{x\to 1^-} f(x)$ exists.	**T**	**F**
$\lim_{x\to 1^+} f(x)$ exists.	**T**	**F**	$\lim_{x\to 1} f(x)$ exists.	**T**	**F**
$f(x)$ is continuous at $x = 0.$	**T**	**F**	$f(x)$ is differentiable at $x = 0.$	**T**	**F**
$\lim_{x\to 3^-} f(x) = 0.$	**T**	**F**	$\lim_{x\to -3^+} f(x) = 1.$	**T**	**F**
$\lim_{x\to -2^+} f(x) = +\infty.$	**T**	**F**	$\lim_{x\to -2^-} f(x) = +\infty.$	**T**	**F**

2. (8 pts. each) Evaluate each of the following limits.

(a) $\lim_{y\to 0} \dfrac{y}{\sin(3y)}$

(b) $\lim_{x\to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

3. (8 pts. each) Let $f(x) = \dfrac{x^2 - 3x}{x^2 - 4}$. Evaluate each of the following limits. If the limit is infinite you must determine whether or not it is $+\infty$ or $-\infty$.

(a) Find $\lim_{x \to -\infty} f(x)$

(b) Find $\lim_{x \to 2^-} f(x)$

(c) Find $\lim_{x \to 0} f(x)$

4. (12 pts.) Find the slope of the tangent line to the graph of the function $f(x) = \dfrac{x}{x-2}$ at the point $(3, 3)$ by calculating the limit of the difference quotient. Then find an equation for the line tangent to the graph there.

5. (12 pts.) Find the derivative $f'(x)$ of the function $f(x) = \sqrt{1+x}$ by using the definition of the derivative as the limit of the difference quotient.