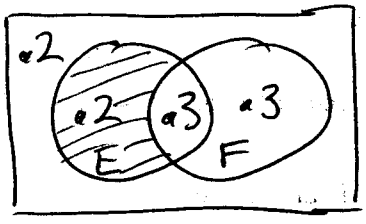
MATH 110 - EXAM 2 - SOLUTIONS

1. (a) $Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$

$\therefore \quad .8 = .5 + .6 - Pr(E \cap F)$

$\therefore \quad Pr(E \cap F) = .3$ //

(b) 

$Pr(E \cap F') = .2$ //

(c) $Pr(E|F) = \dfrac{Pr(E \cap F)}{Pr(F)} = \dfrac{.3}{.6} = .5$

(d) $Pr(F|E) = \dfrac{Pr(F \cap E)}{Pr(E)} = \dfrac{.3}{.5} = .6$

(e) NO. If they were then $Pr(E \cap F) = 0$ which is not true

(f) YES. Independent means $Pr(E|F) = Pr(E)$ which is true (both equal .5).

2. (a) $n(S) = \binom{30}{4}$

$E =$ "all M&Ms are red"

$n(E) = \binom{10}{4}$

$\therefore Pr(E) = \frac{n(E)}{n(S)} = \frac{\binom{10}{4}}{\binom{30}{4}} = \frac{\frac{10!}{4!\,6!}}{\frac{30!}{4!\,26!}} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{30 \cdot 29 \cdot 28 \cdot 27}$

$= \frac{2}{261} \approx .008$

(b) $E =$ "all M&Ms are different colors"

$n(E) = 10 \cdot 10 \cdot 5 \cdot 5$

$\therefore Pr(E) = \frac{10 \cdot 10 \cdot 5 \cdot 5}{\binom{30}{4}} = \frac{10 \cdot 10 \cdot 5 \cdot 5}{\frac{30!}{4!\,26!}} \approx .091$

3. E = "win \$25 at least once during 90 days"

E' = "not win at all during 90 days"

E_1 = "win on day 1" $\quad Pr(E_1) = .01$

E_2 = "win on day 2" $\quad Pr(E_1') = .99$

etc... $\quad$ same for E_2, E_3, etc.

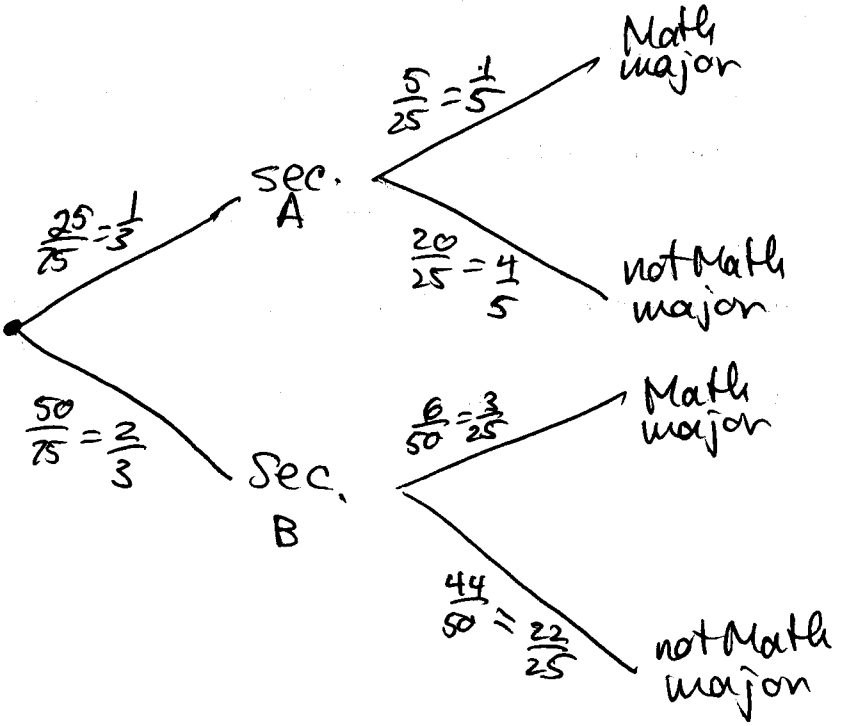
$$\therefore E' = E_1' \cap E_2' \cap E_3' \cap \cdots \cap E_{90}'$$

$$\therefore Pr(E') = Pr(E_1') Pr(E_2') \cdots Pr(E_{90}')$$

$$= (.99)^{90} \approx .405$$

$$Pr(E) = 1 - (.99)^{90} \approx 1 - .405 = .595 //$$

4. (a)



(b) $\Pr(\text{sec A} \cap \text{not Math major}) = \frac{1}{3} \cdot \frac{4}{5} = \frac{4}{15}$

$\Pr(\text{sec B} \cap \text{not Math major}) = \frac{2}{3} \cdot \frac{22}{25} = \frac{44}{75}$

$\therefore \Pr(\text{not Math major}) = \frac{4}{15} + \frac{44}{75} = \frac{64}{75} \approx .853$ //

(c) $\Pr(\text{Math Major} \mid \text{Sec. B}) = \frac{3}{25}$ //

(d) $\Pr(\text{Sec B} \mid \text{Math Major}) = \frac{\Pr(\text{Sec B} \cap \text{Math Major})}{\Pr(\text{Math Major})}$.

$\Pr(\text{Sec B} \cap \text{Math Major}) = \frac{2}{3} \cdot \frac{3}{25} = \frac{2}{25}$

$\Pr(\text{Math Major}) = 1 - \Pr(\text{not Math Major})$

$= 1 - \frac{64}{75} = \frac{11}{75}$

$\therefore \Pr(\text{Sec B} \mid \text{Math Major}) = \frac{\frac{2}{25}}{\frac{11}{75}} = \frac{6}{11}$ //