MATH 108 – QUIZ 3 – 9 FEBRUARY 2011

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (a) (3 pts.) Find the one-sided limits $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$ where $f(x) = \begin{cases} x^2 - x & \text{if } x < 3 \\ 3 - x & \text{if } x \geq 3. \end{cases}$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2 - x) = 3^2 - 3 = 6 //$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (3 - x) = 3 - 3 = 0 //$$

(b) (2 pts.) Is $f(x)$ given in part (a) continuous at $x = 3$? Why or why not?

No. Because $\lim_{x\to 3} f(x)$ does not exist.

2. Let $f(x) = x^2 - 3x$.

(a) (3 pts.) Write down and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$$

$$= \frac{2xh + h^2 - 3h}{h} = \frac{\cancel{h}(2x + h - 3)}{\cancel{h}} = 2x + h - 3$$

(b) (2 pts.) Find $f'(x)$ by computing $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} 2x + h - 3 = 2x - 3 //$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (a) (3 pts.) Find the one-sided limits $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$ where $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 3 \\ 2x + 4 & \text{if } x > 3. \end{cases}$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2+1) = 3^2+1 = 10$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2x+4) = 2 \cdot 3 + 4 = 10$$

(b) (2 pts.) Is $f(x)$ given in part (a) continuous at $x = 3$? Why or why not?

Yes. Because $10 = \lim_{x \to 3} f(x) = f(3) = (3)^2+1$

2. Let $f(x) = \frac{2}{x}$.

(a) (3 pts.) Write down and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \frac{1}{h}\left(\frac{2}{x+h} - \frac{2}{x}\right)$$

$$= \frac{1}{h}\left(\frac{2x - 2(x+h)}{x(x+h)}\right) = \frac{1}{h}\left(\frac{2x - 2x - 2h}{x(x+h)}\right) = \frac{1}{\cancel{h}}\left(\frac{-2\cancel{h}}{x(x+h)}\right) = \frac{-2}{x(x+h)}$$

(b) (2 pts.) Find $f'(x)$ by computing $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-2}{x(x+h)} = \frac{-2}{x^2}$$