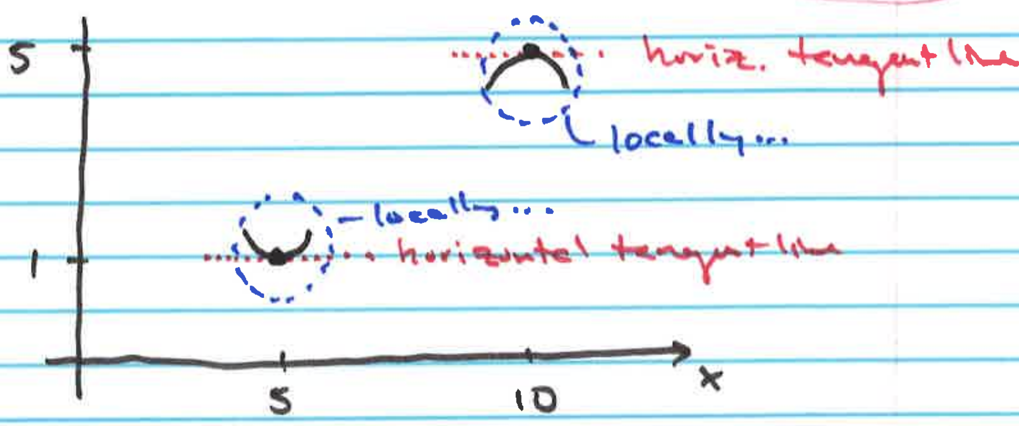
Nov. 5, 2019 ①

4.4 Concavity & Curve Sketching

Suppose we know... f continuous & differentiable

$f(5)=1$, $f'(5)=0$, $f''(5)>0$

$f(10)=5$, $f'(10)=0$, $f''(10)<0$



with this limited information, we know

$$\boxed{\begin{array}{l} f \text{ has a local min at } x=5 \\ f \text{ has a local max at } x=10. \end{array}}$$

<u>Second Derivative Test</u> : Suppose f is continuous near $x=c$.

a) $\boxed{\text{If } f'(c)=0 \ (c = \text{crit. \# of } f) \text{ and } f''(c)>0 \text{ then } f \text{ has a local min at } x=c}$

b) $\boxed{\text{If } f'(c)=0 \ (c = \text{crit \# of } f) \text{ and } f''(c)<0 \text{ then } f \text{ has a local max at } x=c}$

c) $\boxed{\text{If } f'(c)=0 \ (c = \text{crit \# of } f) \text{ and } f''(c)=0 \text{ the 2}^{\text{nd}} \text{ Deriv. Test Provides no information about min/max of } f.}$

EXAMPLE

Find all local min/max values of

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 4$$

2nd Derivative Test

- Need crit. #'s of f

$f'(x) = x^2 - 2x - 3$

$= (x-3)(x+1)$

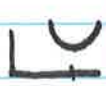
crit #'s $x = 3$, $x = -1$

Note: $f'(3) = 0$ *

$f'(-1) = 0$ *

- Next, compute $f''(x)$

$f''(x) = 2x - 2$



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$f''(3) = 2 \cdot 3 - 2 = 4 > 0$ *

↳ local min of f at $x = 3$

-1

$f''(-1) = 2(-1) - 2 = -4 < 0$ *

↳ local max of f at $x = -1$

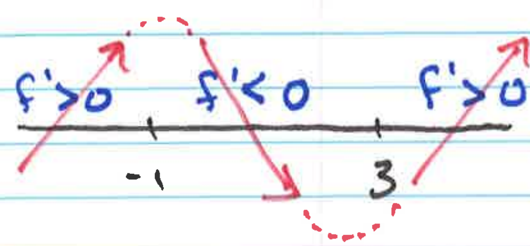
First Deriv. Test

- Need crit #'s of f

$f'(x) = (x-3)(x+1)$

$x = 3$

$x = -1$



So

$\boxed{\text{f has local max at } x = -1}$

$\boxed{\text{f has local min at } x = 3}$

EXAMPLE

Find all local min/max values of

$$f(x) = x^4 + 1$$

Second Der. Test

- crit #'s of f

$f'(x) = 4x^3$

$f' = 0$ when $\boxed{x = 0}$
crit # of f.

- Compute f''

$f''(x) = 12x^2$

$\Rightarrow f''(0) = 0$

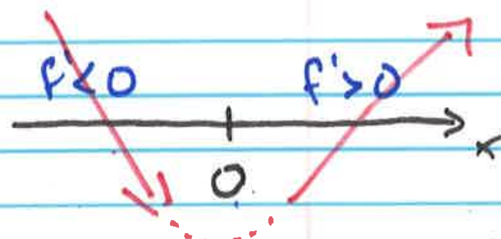
(2nd Der. Test provides no information about min/max at $x = 0$.

First Der. Test

- crit #'s of f

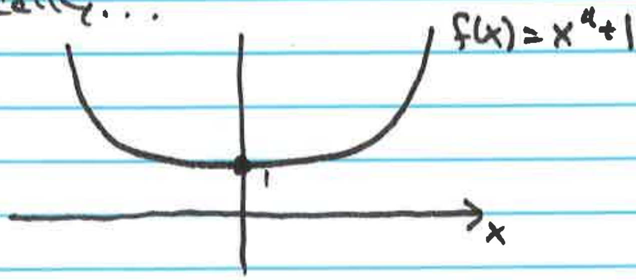
⋮

$x = 0$

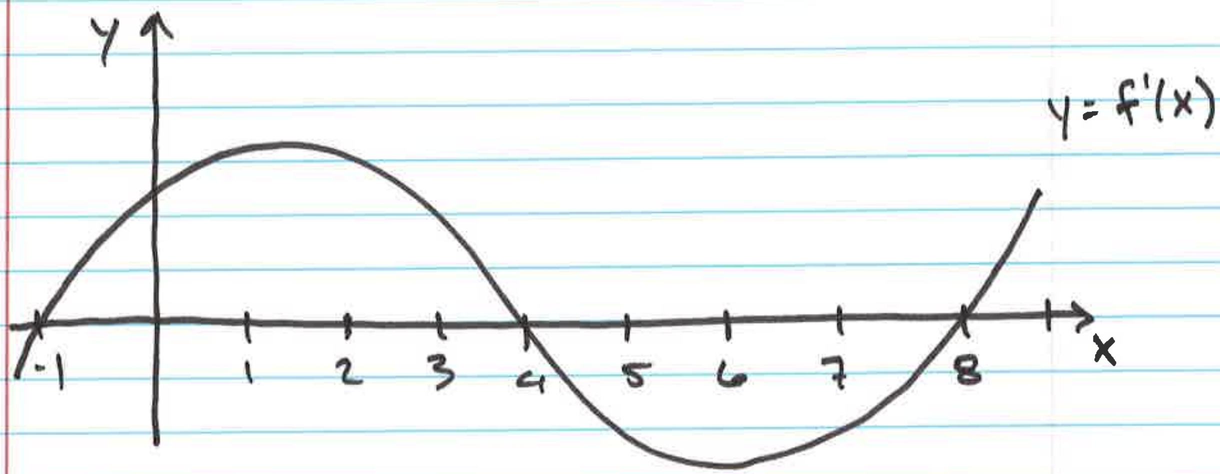


F.D.T. indicates that there is a local min of f at $x = 0$

Basically...



EXAMPLE (Graph of $f'(x)$!!)



a) At what values of x is the function $f(x)$ increasing?

Rephrase: At what values of x is $f' > 0$?

$\boxed{-1 < x < 4}$ AND $\boxed{x > 8}$

b) At what values of x is the function $f(x)$ decreasing?

Rephrase: At what values of x is $f' < 0$?

$\boxed{x < -1}$ AND $\boxed{4 < x < 8}$

c) $\boxed{T}$ or F : $f'(x)$ has a local min at $x = 6$

d) T or $\boxed{F}$: $f(x)$ has a local min at $x = 6$.

note $f'(6) \neq 0$.

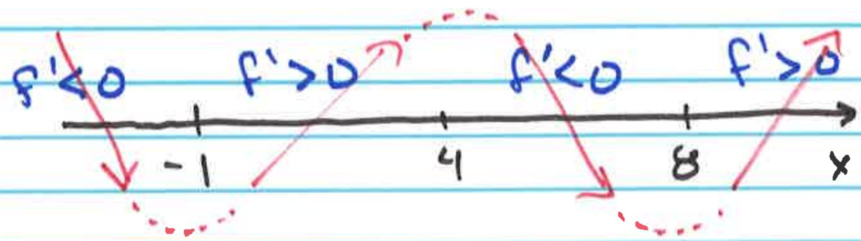
e) ~~Identify the~~ lo

e) Identify the values of x where
f(x) has a local min or local max.

$f'(x)=0$ (crit #'s of f)

when $\boxed{x=-1}$ $\boxed{x=4}$ $\boxed{x=8}$

Try First Deriv. Test



f has local min(s) at $x=-1$ and $x=8$.
f has local max at $x=4$

f) At what values of x is f(x) concave up?

$\boxed{f''(x)>0}$ $\Leftrightarrow$ $\boxed{f'(x) \text{ increases}}$ $\Leftrightarrow$ $\boxed{f \text{ concave up}}$

$\boxed{x<1.5}$ (approximately)

$\boxed{x>6}$

g) At what values of x is f(x) concave down?

$\boxed{f''(x)<0}$ $\Leftrightarrow$ $\boxed{f'(x) \text{ decreases}}$ $\Leftrightarrow$ f conc. down

$\boxed{1.5<x<6}$ *

h) Where are inflection pts. of f?

at $x=1.5$ and $x=6$

i) T or F: $f(2) > f(4)$

False:

note $f'(x) > 0$
between 2 and 4
so f increases from $x=2$ to $x=4$
hence $f(4) > f(2)$.

j) T or F: $f(6) > f(8)$

True: note $f' < 0$ between 6 and 8
so f decreases between
6 and 8: $f(6) > f(8)$.

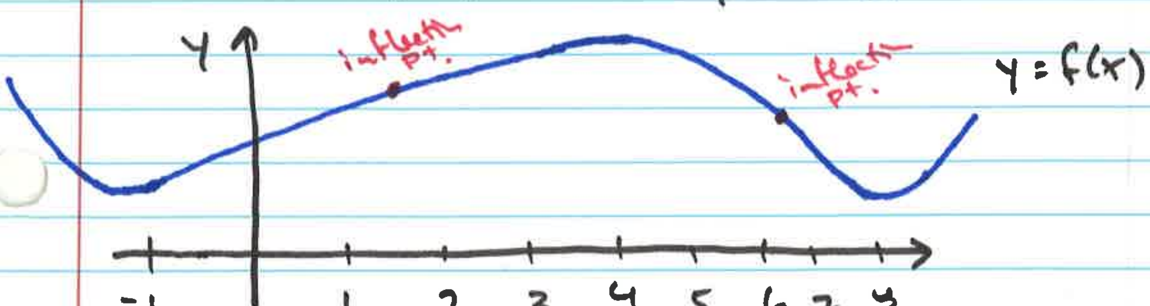
k) T or F: $f'(2) > f'(4)$

from graph...

l) T or F: $f'(6) > f'(8)$

$f'(6)$ negative
$f'(8) = 0$.

What does f(x) actually look like?



f:
local min $x=-1$ $x=8$

local max at $x=4$