Nonlinear Functional Analysis

Math 784-001

Spring 2019

This is the web page **http://math.cos.gmu.edu/~wanner/courses/m784s19/index.html**
It will be updated regularly and always contain the latest information on the course.

General Information:

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Office hours:	T 5-6pm, R 12-1pm, and by appointment

Lectures:	T 7:20-10:00pm, Exploratory Hall 4106
Prerequisites:	Math 675 (Linear Analysis); some familiarity with elementary differential equations.
Textbook:	There is no required textbook for this course.

Important Links:

- Detailed syllabus (including recommended books)
- Homework assignments
- Relevant official GMU policies

Syllabus:

This course covers fundamental techniques in nonlinear functional analysis, as well as selected applications. Topics include the contraction mapping principle, Frechet derivatives and higher derivatives of nonlinear functions between Banach spaces, the implicit function theorem, Lyapunov-Schmidt reduction, Newton polygon method, topological degree theory, and bifurcation theory. A more detailed syllabus can be found here. It will be updated weekly.

Homework Assignments:

Homework problems will be assigned once a week and posted on the homework page. Some of these assignments will be graded and count towards your homework score. While the remaining ones do not have to be handed in, I do advise everyone strongly to study them and write out the solutions properly. I will go through many of the homework problems in the following class and you will not benefit from this if you have not made a serious attempt at solving them. For some of the homework assignments you will need to use the software package Mathematica. Information on how you can obtain Mathematica for your personal computer through the university's site license can be found here.

Grading Policy:

Your final grade in the course will be determined from your performance in the homework assignments and your attendance and class participation. Weights for these items will be distributed approximately according to the following schedule:

Homework	Attendance
90%	10%

Thomas Wanner, January 18, 2019.

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The following table contains the detailed schedule for the course. This page will be updated regularly throughout the semester. Lecture notes for the course will be posted regularly on Blackboard.

Week	Date		Notes
		I. Introduction	
1	01/22	1. Fixed Points and Nonlinear Equations	
		2. Bifurcations of Solutions	
		3. Topological Degree	
		II. Brouwer Degree	
2	01/29	1. The Fundamental Axioms	
		2. First Applications of the Degree	
3	02/05	**No class!**	
4	02/12	3. Fixed Points and Hedgehogs	
		4. Degree Computation	
		5. Two Approximation Results	
5	02/19	6. Construction of the Degree	
		7. Borsuk's Theorem	
6	02/26	8. The Product Formula	
		III. Differential Calculus in Banach Spaces	
		1. Review of Banach and Hilbert Spaces	
7	03/05	2. Linear and Multilinear Mappings	
		3. Gateaux and Frechet Derivatives	
8	03/12	**No class! (Spring Break)**	
9	03/19	4. Partial Derivatives	
		5. Higher Derivatives and Taylor's Formula	
10	03/26	6. The Implicit Function Theorem	
		7. The Newton Polygon	
		IV. Bifurcation Theory	
11	04/02	1. A Necessary Condition for Bifurcation	
		2. Elementary Examples	
12	04/09	3. The Lyapunov-Schmidt Reduction	
		4. Bifurcation from a Simple Eigenvalue	
		5. Application to Euler Buckling	
13	04/16	**No class!**	

V. Topological Degree in Banach Spaces

14	04/23	1. Compact Mappings and Sets
		2. Definition of the Leray-Schauder Degree
		3. Properties of the Leray-Schauder Degree
15	04/30	4. The Degree Calculation
		5. Global Bifurcation

While there is no assigned textbook for the course, I draw material mostly from the following books:

- A. Ambrosetti and D. Arcoya, *An introduction to nonlinear functional analysis and elliptic problems*, Birkhäuser, 2011.
- A. Ambrosetti and G. Prodi, *A primer of nonlinear analysis*, Cambridge University Press, 1993.
- R.F. Brown, *A topological introduction to nonlinear analysis*, Birkhäuser, 1993.
- M. Chipot, *Elements of nonlinear analysis*, Birkhäuser, 2000.
- K. Deimling, *Nonlinear functional analysis*, Dover Publications, 2010.
- Pavel Drabek and Jaroslav Milota, *Methods of Nonlinear Analysis*, Birkhäuser, 2013.
- W. Krawcewicz and J. Wu, *Theory of degrees with applications to bifurcations and differential equations*, Wiley, 1997.
- N.G. Lloyd, *Degree theory*, Cambridge University Press, 1978.
- E. Zeidler, *Nonlinear functional analysis and its applications I*, Springer-Verlag, 1986.

Thomas Wanner, January 22, 2019.

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You can download the assigned homework problems from the following table.

This table will be updated regularly during the semester.

Week	No.	Assigned on	Due on	Homework Assignment
1 (01/22-01/25)	1	01/22	01/29	TBA
2 (01/28-02/01)	2	01/29	02/12	TBA

Thomas Wanner, January 1, 2019.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. It is expected that each student will conduct himself or herself within the guidelines of the Honor Code. All academic work should be done with the level of honesty and integrity that this University demands.
2. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
3. You are responsible for knowing the last days to drop and add this class.
4. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
5. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
6. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
7. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
8. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.
