George Mason University
Department of Mathematical Sciences

Combinatorial Structures: Algebras for Combinatorists

Spring 2019

Course: MATH-723, Combinatorial Structures, section 001.

Total Credits: 3.

Purpose: This topics course will serve as an introduction to the application of abstract algebra in combinatorics. We will discuss some topics (algebras, coalgebras, bialgebras and Hopf algebras) that are not covered in a usual algebra course (group, rings, modules and fields), but have in recent times shown to be useful in combinatorics, such as enumerative combinatorics and graph theory.

The hope is that after this course it should be clear that a good knowledge in commutative algebra, including the concepts of coalgebras and bialgebras, is good for the working combinatorist, just as algebra is a vital tool in topology and functional analysis. At the end of the course we will hopefully have covered enough topics and/or examples for the attendees to be able to read advanced monographs and papers on applications of these algebraic techniques in combinatorics.

Prerequisites: Most important is to have mathematical maturity and an open mind. Roughly what I expect is listed as follows:

- Required – Math 621, Algebra (First year graduate course), or equivalent.
- Preferred – (1) Math 325, Discrete Mathematics II, or equivalent. (2) Math 203, Matrix Algebra (Linear Algebra), or equivalent.

NOTE ! In the GMU catalog `https://catalog.gmu.edu/courses/math/` , click on "700 Level Courses" and scroll down to MATH-723: official prerequisites are only recommended.

Times and Places: TR 1:30 – 2:45 pm. Exploratory Hall, room 4106

Period: From January 22. to May 15.

Dates to keep in mind:

January 29:	Last day to add classes.
February 5:	Last day to drop with 100% tuition refund.
February 12:	Last day to drop with no tuition refund.

Professor:
Geir Agnarsson
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Office-hours: TR 2:45 – 3:45 pm, or by appointment.

Course Text: There is no designated text necessary for this course, but most, if not all, we will cover is contained in the arXiv paper by Grinberg and Reiner `https://arxiv.org/abs/1409.8356`:

- *Hopf Algebras in Combinatorics*, Darij Grinberg & Victor Reiner (last revised 11 May 2018 (version, v5)).

Some material we will discuss is also taken from the following original texts on coalgebras, bialgebras and Hopf algebras:

1. Irving Kaplansky, *Bialgebras*, Lecture Notes in Mathematics, University of Chicago (1975).

2. Moss E. Sweedler, *Hopf Algebras*, Math. Lect. Notes Series, W. A. Benjamin, Inc., New York (1969).

Although the lectures will be mostly self-contained, some results assumed known that are stated without formal proofs will be from the following texts:

1. Thomas W. Hungerford, *Algebra*, Graduate Texts in Mathematics (GTM-73), Springer Verlag, New York (1974), 12th printing (2003) or most recent one.

2. David S. Dummit and Richard M. Foote, *Abstract Algebra*, John Wiley & Sons, Inc., third edition, (2004).

Material: Selected topics from the following: (i) commutative rings, (ii) modules and algebras, (iii) tensor products of modules, (iv) tensor algebras, (v) coalgebras, (vi) bialgebras, (vii) Hopf algebras.

Homework: Homework will be assigned every three weeks or so, a total of four assignments during the semester.

WebSite: All announcements, notes and pdf file handouts for this course will be posted on the following class WebSite:
`http://math.gmu.edu/~geir/courses/723spring19/`

Examinations: There will be no exams in this course.

Grading: Your grade for this course will be based on participation and homework.

Geir Agnarsson
January 22, 2019