

# Math 689 – Dynamics and Stability of Traveling Waves – Spring 2018

**Dates/Times** M 4:30-7:10

**Location** Exploratory 4106

**Instructor** Matt Holzer, Exploratory Hall 4458

**Email** mholzer@gmu.edu

**Office Hours** M 3:00-4:00, W 11:00-12:00, F 2:30-3:30 (others by appointment)

## Resources

- T. Kapitula and K. Promislow. *Spectral and Dynamical Stability of Traveling Waves*, Springer, 2013.
- M. Beck. *Topics in stability theory for partial differential equations*, PhD Thesis, 2006.
- B. Sandstede. *Existence and Stability of Traveling Waves*, online course notes, 2007.
- G. Schneider and H. Uecker. *Nonlinear PDEs: A dynamical systems approach*, AMS, 2017.

**Attendance** Attendance is not an explicit requirement for this course, but as you will see below it is an implicit requirement.

**Recap/Reflection Assignments** Each week a two page reflection is due. The point is to highlight the main concepts and examples studied during the previous week. The goal is not to regurgitate the lecture material, but to try and synthesize it and distill the main ideas. At the beginning of each lecture one student will be called on randomly to present their recap. The recap is worth two points each week and the presentation is worth five.

**Weekly homework assignments** A homework assignment will be distributed every week. These problems will be due the following week. The homework will be divided into two parts. The first will contain short answer, true/false or computer exercises. The second will contain more analytical problems. The first part will be graded on a 1/0 scale. The second portion is graded check/rewrite/zero. A check counts as one point. A rewrite is zero, but the problem may be rewritten and resubmitted for full credit. A zero is a zero.

**Project and Presentation** Each student will study a topic in dynamical systems outside the scope of the class and present on this topic to the class in lieu of a traditional final exam. Topics can be determined by the student or with the aid of the instructor. A one page written project proposal will be required that will outline the topic, the goals of the project and the resources/references to be consulted. Two presentations will be made: first a ten minute presentation meant to introduce the class to the project topic and outline work done to date and future goals. The final presentation will take place on the final exam day and will be 20 minutes/ 5 minutes for questions.

## Important Dates

February 26th: Project Proposal Due

April 2nd: Project Pitch in class

April 30th: Final Presentations

**Grade** Grades will be determined by assigning 70% weight to the aggregate homework and 30% to the project and presentation. Final grades will be given according to the standard breakdown (94 for an A, 90 for an A-, 87 for a B+, etc). I reserve the right to shift these gradelines lower, but they will not be raised.

**Academic Integrity** You are bound by the Mason Honor Code and its policies related to Academic Integrity. Violations will be taken seriously.

**Disability Services** Students may be eligible for accommodations through the Office of Disability Services

**Communication** All email communication is to take place through your gmU email account.

Course outline (tentative)

- January 22: Introduction and motivating examples. Traveling waves and spatial dynamics. ODE and numerical methods review.
- January 29: Linear waves. Fourier and Laplace Transform. Dispersion relations, phase/group velocities and the evolution of wave packets. Dispersion relations for dissipative PDES and Turing bifurcations.
- February 5: Infinite dimensional operators, continuity, boundedness, closed and compact operators. Basics of Spectral Theory. Fredholm operators and Weyl essential spectrum Theorem.
- February 12: Green's functions. Sturm-Liouville Theory for separated and periodic boundary conditions and on the infinite real line. Applications.
- February 19: Essential spectrum for fronts/pulses. Exponential weights. Absolute Spectrum.
- February 26: **Project Proposal Due**
- March 5:
- March 12: **No class: spring break**
- March 19
- March 26:
- April 2: **Project Pitches** Fronts propagating into unstable states. Pulled/pushed fronts and pinched double root criterion
- April 9: Modulation equations: multiscale analysis leading to Ginzburg-Landau and NLS derivations in the neighborhood of bifurcation
- April 16: Eckhaus and Benjamin-Feir instability, eikonal equation
- April 23: Solitons. KdV equation and NLS.
- April 30: **Final Presentations**