

MATH 677 FALL 2017 SYLLABUS Prof. Sachs

Welcome to Math 677! Math 677 is a first graduate-level course on Ordinary Differential Equations. The material is some of the most profound and most useful parts of all mathematics. The history of differential equations probably begins with ancient astronomy and continues to modern space exploration, along with many other applications. It took a long period of time and the contributions of many individuals (some geniuses and many non-geniuses) to reach its present form. Differential equations has major impact on other areas of advanced mathematics also: beyond the obvious link to partial differential equations, ODEs arise in the differential geometry of shortest paths on manifolds, on deformations used in algebra and geometry, on the theory of Lie groups, and as motivating examples in the theory of linear operators and functional analysis.

Course Prerequisites: We presume a prior undergraduate differential equations course. This includes the basic notions of what is a differential equation, methods for solving first-order scalar equations, linear second-order equations, and constant coefficient systems using eigenvalues and eigenvectors. As these come into play, reminders and refreshers will be given, but only briefly. You are responsible outside of class for meeting with me or finding other resources to fill in gaps in your background.

Class Meetings: We meet Wednesdays, from 16:30 to 19:10 in EXPL 4106.

Textbook: Our textbook is *Differential Dynamical Systems (Revised Edition)* by James D. Meiss (published by SIAM). We will cover Chapters 1-6 with additional material from Chapter 8 mainly as time permits.

Software: We will be using *Mathematica* and/or *MATLAB* software, which are free for students under Mason's site license. I will provide detailed information on how to access the software using our computer labs, both actual and virtual.

Grading: Grading will be fair and impartial. It is based on a mixture of graded (mostly) weekly homework (40%), one mid-term exam (30%) and a final exam (30%). Grades are meant to reflect your apparent achievement of understanding and competency, which is not based directly on the achievements nor lack of achievements of your classmates. Your high achievement will not "blow up the curve" for anyone else. The grade distribution at the end will be what it is; there is no preset distribution of competency nor is competency distributed randomly, so we do not necessarily expect a bell curve.

Gifts: None will be given as grades. **If you need or want a particular grade, you are responsible for earning it.** I will work with you to achieve your goal.

Exam Dates:

- Mid-Term Exam: Wednesday, October 11 during class
- **Final: Wed. 12/13 4:30 pm 7:15 pm**

Policies: The GMU Honor code is in effect at all times and students are expected to be fully aware of its requirements. Group work may be part of the course and group members will truthfully report on non-contributing members. **Absence from exams must be for a valid reason and requires prior notification except in extreme circumstances.** Students are encouraged to talk with one another, but work should be theirs. **DO NOT TURN IN SOLUTIONS OBTAINED FROM ONLINE SOURCES.**

Office Hours: Room 4211 in Exploratory Hall, on Wednesdays from 2:45 to 4:15pm. I can also meet with students at other times as needed. If you show up at my door at some random time with no appointment, I may be unable to see you then due to other work.

Contact Info: My email is rsachs@gmu.edu and my office phone is 703-993-1464. Use headers on email to avoid spam filters.

Schedule of Topics: Our pace is not leisurely in this course. Mathematics courses are inherently cumulative, so later topics often use earlier material in a new setting. **DO NOT FALL BEHIND EARLY.** Roughly the plan is:

- Week 1: Prior ODE brief review; Introduction on modeling and examples (textbook Sections 1.1-1.7)
- Week 2: Linear Systems I (textbook 2.1-2.5)
- Week 3: Linear Systems II (textbook 2.6- 2.8)
- Week 4: Existence and Uniqueness with Preliminaries (textbook 3.1-3.3)
- Week 5: Dependence on Initial Conditions; Maximal Interval of Existence (textbook 3.4-3.5)
- Week 6: Dynamical Systems concepts: definitions, flows, linearization (textbook 4.1-4.4)
- Week 7: Mid-Term Exam; Stability and Lyapunov Functions (textbook 4.5-4.6)
- Week 8: Topological Conjugacy; Hartman-Grobman Theorem; Omega-limit sets (textbook 4.7-4.9)

- Week 9: Attractors and their Basins; Stability of Periodic Orbits; Poincaré maps (textbook 4.10-4.12)
- Week 10: Invariant Manifolds I (textbook 5.1-5.3)
- Week 11: Invariant Manifolds II (textbook 5.4- 5.6)
- Week 12: Phase Plane I (textbook 6.1-6.4)
- Week 13: Phase Plane II (textbook 6.5-6.7)
- Week 14: Introduction to Bifurcation. A glimpse at further topics. Final review. (textbook 8.1-8.6 perhaps)

Caveat on schedule: The schedule may be amended as needed. If we end up faster, this allows some additional interesting topics.

A bit about me: I found undergraduate differential equations to be among my favorite courses and went to graduate school intending to study more on this. I did my doctoral work at the Courant Institute, writing a PDE thesis using ideas from dynamical systems in part. I was a postdoc at Wisconsin, then was faculty at Penn State prior to coming to Mason. I was department chair here also.