NEW CRITERIA FOR BOUNDEDNESS AND COMPACTNESS OF WEIGHTED COMPOSITION OPERATORS MAPPING INTO THE BLOCH SPACE

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An interesting question in operator theory is:

Given Banach spaces X and Y and a linear operator $T : X \to Y$, what is a minimal collection of functions in the range of T whose boundedness in norm in Y guarantees the boundedness of T?

In this talk, we study this problem in the case of the weighted composition operator

$$W_{\psi,\varphi} : f \mapsto \psi(f \circ \varphi) \ (\psi, \varphi \text{ analytic on } \mathbb{D},\ \varphi(\mathbb{D}) \subseteq \mathbb{D})$$

from several classical functional Banach spaces into the Bloch space $\mathcal{B}$, defined as the space of the analytic functions f on the open unit disk $\mathbb{D}$ in the complex plane such that

$$|f'(z)| = O\left(\frac{1}{1-|z|^2}\right), \text{ for } z \in \mathbb{D}.$$

Furthermore, we obtain new compactness criteria in terms of the little "oh"-condition on the Bloch norm of suitable collections of functions. We show that boundedness and compactness of such operators can be formulated in terms of at most two one-parameter families of functions and, in some cases, one of the families is a countable collection.

Part of this research was motivated by the wish to generalize to weighted composition operators the following compactness criterion for composition operators on the Bloch space proved by Wulan, Zheng and Zhu:

C_φ is compact on $\mathcal{B}$ if and only if $\lim_{n\to\infty} \|\varphi^n\|_{\mathcal{B}} = 0$, where $\|f\|_{\mathcal{B}}$ is the Bloch norm of f.