Characterizing Witnesses to the Non-normality of $N^{\aleph_1}$ and the Necessity of Arcs in Some Rim-finite Spaces.

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April 29
At 1pm
ST1 Room 120

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A submission for: Doctorate of philosophy in mathematics

Department name: Mathematics

Abstract

With regard to the non-normality of $N^{\aleph_1}$, we study the properties a closed set $Z \subset N^{\aleph_1}$ holds both as a topological space and relatively as a subspace of $N^{\aleph_1}$, that guarantee the existence of a closed set Z' where Z' is disjoint from Z and fails to have an open separation with Z. In part we show that for a closed set Z there exists a closed set Z' disjoint from Z and having no open separation with Z if and only if there exists an uncountable closed discrete set D where D is disjoint from Z and fails to have an open separation with Z. We also among other things show that for every closed set Z with a non-Lindelöf boundary there exists a countable closed discrete set A where A and Z are disjoint and fail to have an open separation. We use the preceding results to generate new and pathological witnesses to the non-normality of $N^{\aleph_1}$. Among the new witnesses we include a pair of disjoint countable closed discrete sets which fail to have an open separation with disjoint open sets having disjoint closures.

With regard to rim-finite spaces, in order to show that an example of an arc free 1-dimensional rim-3 subspace of the plane given by J. Kulesza is optimal, we show that every 1-dimensional separable rim-2 space must include an arc. The fact that all rim-1 spaces are zero dimensional yields that from the inclusion of arcs in separable 1-dimensional rim-2 spaces, it may be concluded that not only is J. Kulesza's example of an arc free 1-dimensional rim-3 subspace of the plane the smallest known example of such a space but is also the smallest possible example of such a space.