1. Define Field Axioms, Positivity Axioms and Completeness Axiom.

2. Prove, directly from the axioms above, the following properties of real numbers: (i) if $a > 0, c < 0$, then $ac < 0$; (ii) if $a > 0, b > 0$ and $a < b$, then $1/a > 1/b$; (iii) there exists a positive real number $a$ such that $a^2 = 2$.

3. Let $\{a_n\}$ be a sequence of real numbers and $a$ a real number. Define $\lim_{n \to \infty} a_n = a$.

4. Prove, directly from the definition, that $\lim (2n^2 - 4n - 100)/(n^2 - 16n + 2) = 2$.

5. Prove that $\lim_{n \to \infty} a_n = a$ if and only if for every $\epsilon > 0$, the set $\{n | n \in \mathbb{N}, |a_n - a| \geq \epsilon\}$ is finite.

6. Using problem #4, prove that a sequence $\{a_n\}$ does not converge to $a$ if and only if there exists $\epsilon > 0$ such that $|a_n - a| \geq \epsilon$ for infinitely many natural numbers $n$.

7. Prove that $||a| - |b|| \leq |a - b|$ for any real numbers $a, b$.

8. Prove, directly from the definitions, that every convergent sequence is bounded.

9. Prove, directly from the definitions, that if $\lim_{n \to \infty} a_n = a, \lim_{n \to \infty} b_n = b$ and $c$ is a real number, then $\lim_{n \to \infty} a_n + b_n = a + b$, $\lim_{n \to \infty} c a_n = ca$, and $\lim_{n \to \infty} a_n b_n = ab$.

10. Prove that a number $b$ is the sup of a bounded set $B$ if and only if $b$ is an upper bound of $S$ and for every $\epsilon > 0$, there exists a number $x \in B$ such that $x > b - \epsilon$.

11. Prove that if $\{a_n\}$ is an increasing sequence that is bounded, then $\{a_n\}$ converges to sup$\{a_n\}$.

12. Define $a_1 = 1, a_n = 1/(1 + a_{n-1})$ for $n \geq 2$. Prove that the sequence $\{a_n\}$ converges. Find the limit of the sequence.

13. Find a convergent subsequence of the sequence $\{(-1)^n\}$.

14. Prove that every sequence has a monotone subsequence.

15. Prove that every bounded sequence has a convergent subsequence.

16. Let $D$ be a set of real numbers and $f : D \rightarrow \mathbb{R}$ a function. Let $x_0 \in D$. Define continuity of $f$ at $x_0$.

17. State the Extreme Value Theorem.


19. Try to do ALL assigned practice problems.