Unless otherwise stated, Euclidean space is assumed in each question.

1. Prove that in a cyclic quadrilateral, the product of the lengths of the two diagonals is equal to the sum of the products of lengths of the opposite sides.

2. In Fig. 1, $ABC$ is an arbitrary triangle, $DAB, EAC, FBC$ are equilateral triangles. Prove that $CD, BE, AF$ concurrent at a point $P$, and that all angles at $P$ are $60^\circ$.

3. (continued) Prove that the circumcenters of $DAB, EAC, FBC$ form an equilateral triangle.

4. Let $H$ be the orthocenter of a triangle $ABC$. Prove that for any point $P$ on the circumcircle of $ABC$, the midpoint of $HP$ is on the nine-point circle of $ABC$.