Review problems Math 301

1. Find the prime factorization of 1800.

2. Use your answer to problem 1 to find \(\phi(1800)\), \(\tau(1800)\) and \(\tau(1800)\).

3. Carefully define the following, using complete sentences:
   (i) \(a \equiv b \pmod{m}\) where \(m\) is a positive integer and \(a\) and \(b\) are any integers.
   (ii) An inverse of an integer \(a\) modulo \(m\), where \(m\) is a positive integer.
   (iii) The Chinese Remainder Theorem.
   (iv) Wilson's Theorem
   (v) Fermat's Little Theorem

4. Prove that if \(2^m - 1\) is prime then \(m\) is prime. Give an example to show that the converse is false.

5. Prove that if \(2^m + 1\) is prime then \(m\) must be of the form \(2^k - 1\). Give an example to show that the converse is false.

6. Find the, without a calculator, the gcd of \(2^{18} - 1\) and \(2^{24} - 1\)

7. Find an inverse of 7 modulo 9. Then use your answer to solve \(7x \equiv 3 \pmod{9}\)

8. Solve \(4x \equiv 3 \pmod{18}\)

9. Solve \(4x \equiv 18 \pmod{8}\)

10. Solve \[
    \begin{cases}
    x \equiv 2 \pmod{25} \\
    x \equiv 7 \pmod{9}
    \end{cases}
    \]

11. Solve \[
    \begin{cases}
    x \equiv 2 \pmod{6} \\
    x \equiv 4 \pmod{5} \\
    x \equiv 6 \pmod{7}
    \end{cases}
    \]

12. Find the remainder when \(7^{10}\) is divided by 45.

13. Find the remainder when \(21^{710}\) is divided by 225.

14. Decrypt the letters AC assuming the affine code \(C = 7P + 3\) was used to encode the letters.

15. Prove that if \(p\) is an odd prime, then \(2(p-3)! \equiv -1 \pmod{p}\).
16. Verify that 0-13-011690-4 is a valid ISBN.

17. Which of the following numbers are divisible by 13?

   (a) 78, 918, 239, 735
   (b) 1,086, 320, 015
   (c) 2,086, 320, 015

18. Which of the following numbers are divisible by 11?

   (a) 8,924, 310, 064, 537
   (b) 1,086, 320, 015

19. Which of the following numbers are divisible by 3? by 4?

   (a) 7,245, 670, 124
   (b) 9,316, 279, 014

20. Find the remainder when 50! is divided by 43.