1. 1, 2, 3, 5, 8

2. \[ n = 0 \quad \text{and} \quad 1 + nh = 1 \leq (1+h)^n = 1 \quad \checkmark \]
   
   Assume: \[ 1 + kh \leq (1+h)^k \]

   \[ n = k+1 \]
   \[ (1+h)^{k+1} = (1+h)^k (1+h) \]
   \[ \geq (1+kh) (1+h) \]
   \[ = 1 + kh + kh^2 \]
   \[ = 1 + (k+1)h + kh^2 \]
   \[ \geq 1 + (k+1)h \]
   \[ \therefore \quad kh^2 \geq 0 \]

   \[ \therefore 1 + nh \in (1+h)^n \quad \forall \text{ integer } n \geq 0. \]

3. \[ 7 \frac{15}{160} \text{ in } 110 = 7 \cdot 15 + 5 \]
   \[ \frac{7}{32} \text{ in } -110 = 7 \cdot (-15) - 5 \]
   \[ 32 = 7 \cdot (-16) + 2 \]

4. \[ a = 2 \cdot 3 \cdot 7 \cdot 11^2 \]
   \[ b = 2^3 \cdot 5 \cdot 7^2 \cdot 13 \]

   \( (a,b) = 2 \cdot 7 \)

   \[ [a,b] = 2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \]

5. \[ 65 = 39 \cdot 1 + 26 \]
   \[ 39 = 26 \cdot 1 + 13 \]
   \[ 26 = 13 \cdot 2 \]

   \[ \therefore \gcd = 13 \]

   \[ 13 = 39 - 26 \cdot 1 \]
   \[ = 39 - (65 - 39) \]
   \[ = 39 - 65 \]
   \[ = 9 \cdot 2 \cdot 39 + (-1) \cdot 65 \]

   \[ 13 = \text{ remainder} \]
6. \[ 8388 = 2 \cdot 4194 \]
   \[ = 2 \cdot 2 \cdot 2097 \]
   \[ = 2 \cdot 3 \cdot 699 \]
   \[ = 2 \cdot 3 \cdot 233 \]
   233 is prime by the given fact.

7. \[ 12x + 33y = 132 \]

\[ \div 3 : 4x + 11y = 44 \]

By inspection \( x = 0, y = 4 \) is a solution.

General sol. is

\[ x = 0 + 11k \]
\[ y = 4 - 4k \]

\[ x \geq 0, y \geq 0 \Rightarrow x \geq 0, 4 - 4k \geq 0 \]
\[ x = 0, 4k \leq 4 \Rightarrow k \leq 1 \]
\[ 0 \leq k \leq 1 \Rightarrow k = 0, 1 \]

\[ k = 0 : x = 0, y = 4 \quad (Ans.) \]
\[ k = 1 : x = 11, y = 0 \]

8. Suppose on the contrary that \( a_i > b_i \) for some \( i \), say
\[ a_i = b_i + k \]
where \( k > 0 \).

Then \[ \frac{b_{a_i}}{p^{a_i}} = \frac{p_i^{b_i} \cdots p_n^{b_n}}{p_i^{b_i + k}} = \frac{p_i^{b_i} \cdots p_i^{b_n} p_i^k}{p_i^{b_i} \cdots p_i^{b_n}} \quad (1) \]

Since \( p_i, \ldots, p_n \) are distinct primes, (1) cannot be an integer. So \( p_i^{a_i} \nmid b \). But \( p_i^{a_i} \mid a \) and \( a \mid b \) \( \Rightarrow \)
\( p_i^{a_i} \mid b \), a contradiction.

Therefore we must have \( a_i \leq b_i \) for every \( i \).