Review Homework 4  
Math 213

Do the sample final exam on my webpage, plus the following:

1. Find an equation of the plane through (0,1,2), (3,1,4) and (-1,-5,2).

2. Find an equation of the plane containing the line
   \[
   \frac{x-2}{2} = \frac{y}{2} = \frac{z-1}{4}
   \]
   and the point (1,2,3).

3. Sketch the surface (a) \( z = x^2 + y^2 \), (b) \( z = \sqrt{x^2 + y^2} \).

4. If a point has rectangular coordinates \((\sqrt{2}, -1, -5)\), find its spherical coordinates.

5. If the temperature at any given point \((x,y,z)\) is \(2x^2 + 2y^2 + 3z^2\), find the direction in which the temperature changes most rapidly at the point \((2,1,3)\).

6. If the temperature at any given point \((x,y,z)\) is \(\frac{2}{x^2 + 2y^2 + 3z^2}\), find the rate of change of the temperature in the direction of the vector \(<1, 1, -1>\) at the point \((2,1,3)\).

7. Evaluate the integral
   \[
   \int \int \int_E \sqrt{x^2 + y^2 + z^2} \, dV
   \]
   where \(E\) is the solid bounded by the cylinder \(x^2 + y^2 = 1\) and the two planes \(z = 1\) and \(z = 2\).

8. Find the local maximum and minimum values and saddle points of the functions: a. \(f(x,y) = x^2 - 6xy + 8y^3\)  
   b. \(f(x,y) = (x^2 + y)e^{x/2}\).

9. Find the absolute maximum and minimum values of  
   a. \(f(x,y) = y^2 - xy^3\) on the closed triangular region in the \(xy\)-plane with vertices \((0,0),(0,6),\) and \((6,0)\).  
   b. \(f(x,y) = e^{-x^2}(x^2 + 2y^2)\) on the disk \(x^2 + y^2 \leq 4\).
10. (optional) Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the vector field

$$\mathbf{F}(x, y, z) = xi + yj - zk$$

and the curve $C$ is the line $x = 1 + t, y = 2 - t, z = t, 0 \leq t \leq 2$.

11. (optional) Use Green’s theorem to evaluate $\int_C \mathbf{f} \cdot d\mathbf{r}$, where $C$ is the circle $x = 2 \cos t, y = 2 \sin t, 0 \leq t \leq 2\pi$ and $\mathbf{f}(x, y) = xi + 2yj$. 