Review Problems for Chapter 1
August 30, 2001

1. Does the following rule define a function? Why?
   \[ f(x) = \begin{cases} 
   -x + 1 & \text{for } x \leq 1 \\
   x^2 - 1 & \text{for } x \geq -1 
   \end{cases} \]

2. Graph the following function:
   \[ f(x) = \begin{cases} 
   -x + 1 & \text{for } x \leq 1 \\
   x^2 - 1 & \text{for } x > 1 
   \end{cases} \]

3. Graph (a) \(2f(x - 1) - 3\), (b) \(f(-2x)\), (c) \(-f(x/2)\), where the graph of \(f\) is given on the page:

4. Graph (a) \(2f(x - 1) - 3\), (b) \(f(-2x)\), (c) \(-f(x/2)\), where \(f(x) = \sqrt{x}\).

5. Find a formula for the inverse of the function
   \[ f(x) = \frac{1 + e^x}{1 - e^x} \]

6. Give an example of an one-to-one function \(f(x)\) such that \(f^{-1}(x) \neq \frac{1}{f(x)}\).

7. If \(f(x) = \ln x\) and \(g(x) = x^2 - 9\), find the functions \(f \circ g, g \circ f, f \circ f, g \circ g\), and their domains.

8. Solve the equation for \(x\): \(2^{100} = 10^x\).

9. If \(f(x) = 2x + \ln x\), find \(f^{-1}(2)\).

10. A rectangle has area 36 \(m^2\). Express the perimeter of the rectangle as a function of the length of one of its sides.

11. Express the surface area of a cube as a function of its volume.

12. A rectangular storage container with an open top has a surface area of 10 \(m^2\). The length of its base is twice its width. Material for the base costs $10 per square meter; material for the sides costs $6 per square meter. Express the cost of materials as a function of the width of the base.

13. Express the function \(\sqrt{x^2 + 1}\) as a composition of two functions.