Consider the IVP $\ddot{x} + 4x = \delta(t - 2\pi) - \delta(t - 4\pi)$, $x(0) = 0$, $\dot{x}(0) = 0$.

1. (a) (20 pts) Solve the IVP by Laplace transforms. Simplify your answer.
   (b) (10 pts) Sketch a graph of the solution. Be sure to label all significant coordinates.

2. A linear system has transfer function $K(s) = (s + 1)/(s^2 + 2s + 2)$.
   (a) (10) Determine $k(t) = \mathcal{L}^{-1}[K(s)]$.
   (b) (10) If $e^{-\pi s}/(s + 1)$ is the LT of the input $f(t)$, determine $f(t)$.
   (c) (10) Compute the output (solution) of the system.

3. The motion of an undamped pendulum is given by the equation $\ddot{x} + 4x = 0$.
   (a) (5 pts) Express the 2nd-order ODE as a pair of 1st-order ODEs.
   (b) (5 pts) What is the 1st-order ODE for the orbits in the $xy$-plane? (Take $y = \dot{x}$.) Do not solve!
   (c) (5 pts) Now solve the ODE you obtained in part (b).
   (d) (10 pts) Sketch the phase portrait for the solution with initial values $x(0) = 0$, $y(0) = 2$. Be sure to label axes and any important coordinates. Indicate the direction of motion on the curve.
   (e) (5 pts) What is the matrix $A$ when the pair of 1st-order ODEs in part (a) is expressed in the vector form: $\dot{x} = Ax$.
   (f) (5 pts) Convert the initial values of part (d) to an initial vector $x_0$ for the system $\dot{x} = Ax$. What is $x_0$?
   (g) (5 pts) Compute the eigenvalues of $A$.
   (h) (10 pts) Compute the eigenvectors corresponding to each of the eigenvalues of $A$.
   (i) (10 pts) Determine a basis of real vector solutions for the system $\dot{x} = Ax$.
   (j) (10 pts) Solve the IVP, $\dot{x} = Ax$, $x(0) = x_0$, where $x_0$ was formed in part (f).