Name: ANSWER KEY

Math 554-B01, Summer 2014, Test 3, O’Beirne

Answer all questions. Show all work leading to the answer for credit. You must work alone. The Honor Code is in effect.

1. A ten-year $1,000 par value bond bearing a 10% coupon rate payable semi-annually and redeemable at $1,100 is bought to yield 8% compounded semi-annually. Find the purchase price.

Work:

\[
\begin{align*}
PMT &= 50 \\
FV &= 1100 \\
i &= 4\% \\
m &= 20 \\
CPT: PV &= 1115.47
\end{align*}
\]

2. An n-year $1,000 par value bond has a redemption value of $1,000 and has a coupon rate of 12% compounded semi-annually. It is bought at a price to yield 10% compounded semi-annually. If the term of the bond is doubled, the price will increase by $48. Find the price of this n-year bond.

Work:

\[
P = 1200 + (1000 - 1200) v^n \\
= 1200 - 200v^n \\
P + 48 = 1200 - 200v^{2n} \\
\therefore 1248 = 1200 - 200v^{2n} \\
200v^{2n} - 200v^n + 48 = 0 \\
v^n = .6 \\
\therefore P = 1200 - 200v^n = 1080 \ \text{or} \ 1120
\]

3. For a $1 bond the coupon rate is 120% of the yield rate and the premium is p. For another $1 bond with the same number of coupons and the same yield rate, the coupon rate is 60% of the yield rate. Express the price of the second bond as a function of p.

Work:

\[
P_1 = 1 + p = 1 + (1.2i - i) a_{\frac{1}{2}} = 1 + .2i \bar{a}_{\frac{1}{2}} \\
\rightarrow p = .2i \bar{a}_{\frac{1}{2}} \\
P_2 = 1 + (.6i - i) a_{\frac{1}{2}} = 1 - .4i \bar{a}_{\frac{1}{2}} = 1 - 2p
\]
4. A $100 bond with annual coupons is redeemable at par at the end of 15 years. At a purchase price of $90 the yield rate is exactly 1% more than the coupon rate. Find the yield rate of the bond.

Work:

\[ q_0 = 100 \left[ 1 - .01 \frac{a_{15}}{i} \right] \]
\[ \rightarrow \frac{a_{15}}{i} = 10 \]
\[ \rightarrow i = 5.5565\% \text{ by calculator} \]

(\( \text{PMT}=1 \quad N=15 \quad PV=-90 \text{ CPT}\ i \))

5. A $1000 par value 10% bond with quarterly coupons is callable five years after issue. The bond matures for $1000 at the end of ten years and is sold to yield a nominal rate of 8% compounded quarterly under the assumption that the bond will not be called. Find the redemption amount at the end of five years that would provide the purchaser the same yield rate.

Work:

\[ P = 1000 \left( 1.02 \right)^{-40} + 25 \frac{a_{40 | 2\%}}{2^{\%}} = 452.89 + 683.89 = 1136.78 \]
\[ 1136.78 = C \left( 1.02 \right)^{-20} + 25 \frac{a_{20 | 2\%}}{2^{\%}} \]
\[ \rightarrow C = 1136.78 \left( 1.02 \right)^{20} - 25 \frac{S_{20 | 2\%}}{2^{\%}} = 1649 - 607.43 = 1081.77 \]

6. A common stock pays annual dividends at the end of each year. The earnings per share in the year just ended were $8. Earnings are expected to grow at the rate of 10% each year in the future. The percentage of earnings paid out as a dividend will be 0% for the next 4 years and 50% thereafter. Find the theoretical price of the stock that would yield the investor 12% effective.

Work:

\[ \frac{\text{EPS}}{2(1.1)^{1}} \quad \frac{\text{EPS}}{2(1.1)^{2}} \quad \frac{\text{EPS}}{2(1.1)^{3}} \quad \frac{\text{EPS}}{2(1.1)^{4}} \quad \frac{\text{EPS}}{2(1.1)^{5}} \quad \frac{\text{EPS}}{2(1.1)^{6}} \cdots \]

\[ PV = V^5 \left( \frac{8(1.1)^5}{.50} \right) + V^6 \left( \frac{8(1.1)^6}{.50} \right) + \cdots \]
\[ = V^5 \left( \frac{8(1.1)^5}{.50} \right) \left( \frac{1}{1 - V(1.1)} \right) = \frac{.5674 \left( 6.44204 \right)}{.017857} = 204.69 \]
7. An investor makes a single deposit of $20,000 into Fund A for 10 years which earns 6% effective rate of interest payable directly to the investor each year. During the first 5 years the interest payments can only be reinvested into Fund B which earns 5% effective over the course of the ten years. During the second five years the interest payments can only be reinvested into Fund C which earns 4% effective. Find the total accumulated value in Funds A, B, and C combined at the end of ten years to the nearest dollar. Find the overall yield rate achieved by the investor.

Work:

\[ 20,000 \times 12 \times \frac{1}{5} \times (1.05)^5 + 1200 \times \frac{1}{5} 
= 20,000 + 6630.76 (1.05)^5 + 6499.59 
= 34,962.31 
\]

\[ 20,000 (1+i)^{10} = 34,962.31 \rightarrow i = 5.744\% \]

8. A loan of $10,000 is being repaid with payments of $1,000 at the end of each year for 25 years. If each payment is reinvested at 6% effective, find the effective annual rate of interest earned over the 25-year period.

Work:

\[ \frac{10000}{\frac{1}{25} \times 6\%} = 54,864.51 \]

\[ 10,000 (1+i)^{25} = 54,864.51 \]

\[ (1+i)^{25} = 54,864.51 \]

\[ 1+i = 1.07462 \]

\[ 7.05\% \]

9. A $100 par (face) value 12-year bond with 8% semiannual coupons is selling for $120. If the coupons can only be reinvested at 6% compounded semiannually, compute the overall yield rate achieved by a bond purchaser over the 12-year period.

Work:

\[ 4 \times \frac{1}{4} \times 3\% + 100 
= 137.71 + 100 = 237.71 
\]

\[ 120 (1+j)^{24} = 237.71 
(1+j)^{24} = 1.9809 
1+j = 1.02849 
\]

\[ j = 0.02849 
\]

\[ i = 2(0.02849) = 0.05778 \]

\[ 5.778\% \]
10. On January 1 an investment account is worth $100,000. On March 1 the value has increased to $110,000 and $25,000 of new principal is deposited. On September 1 the value has declined to $125,000 and $40,000 is withdrawn. On January 1 of the following year the account is again worth $100,000. Calculate the yield rate by the dollar-weighted method and by the time-weighted method.

\[ i_{DW} = 13.95\% \quad i_{TW} = 19.83\% \]

Work:

\[ I = B - A - C = 100 - 100 - (-15) = 15 \]

\[ \frac{15}{100(1) + 25\% - 40(\frac{1}{3})} = \frac{15}{100 + 20.83 - 13.3} = \frac{15}{107.5} = 13.95\% \]

\[ \frac{110}{135} \times \frac{100}{85} - 1 = 19.8257 = 19.8257\% \]

11. Use the following table excerpt to find the dollar amount of interest which $10,000 invested at time \( z \) will earn over the 3rd through 6th years of investment (inclusive).

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Investment Year Rates</th>
<th>Portfolio Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Of Investment</td>
<td>( i_1 ) ( i_2 ) ( i_3 ) ( i_4 )</td>
<td>(over and down)</td>
</tr>
<tr>
<td>( Z )</td>
<td>8%  8.10% 8.20% 8.25% 8.20%</td>
<td></td>
</tr>
<tr>
<td>( Z+1 )</td>
<td>9%  9.10% 9.20% 9.25% 9.90%</td>
<td></td>
</tr>
<tr>
<td>( Z+2 )</td>
<td>10% 10.10% 10.20% 10.25% 8.75%</td>
<td></td>
</tr>
</tbody>
</table>

Work:

After 2 years: \( 10,000(1.08)(1.081) = 11,674.80 \)

After 6 years: \( 11,674.80(1.082)(1.0825)(1.082)(1.085) = 16,053.20 \)

Difference: \( 16,053 - 11,675 = 4,378 \)

12. Payments of $100 now and $110 two years from now are equivalent to $209.80 one year from now at either rate \( i \) or \( j \). Find the absolute difference of the two rates.

Work:

\[ 100 + 110v^2 = 209.8 \]

Solve for \( v \):

\[ v = \frac{209.8}{100 + 110} \]

\[ \frac{12 - i_2}{i_2} = .04 \]