George Mason University Math Club problems  
Year 1, #1

Oct. 20, 2004

1.1.1) Evaluate
\[ \sum_{n=1}^{\infty} \frac{d(n)}{n^2} \]
where \( d(n) \) gives the number of positive integers that evenly divide \( n \). (Hint: recall that \( \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \)).

1.1.2) Come on down! You’ve just made it on the Price is Right. In fact, you’ve just won A New Car! You’ve gotten a taste of winning, and you really like it. You’re at the big wheel, and you’d really like to make it to the final round of the game. So much, in fact, that you wonder what the optimal strategy is for each player, and what the probability of winning is for each player if they all follow their optimal strategies.

**Notes:**

1. For the benefit of those who do not watch, three players compete at the big wheel. The wheel has 20 spaces from $0.05 to $1.00 in five cent increments. Each player spins once, then has the option of either spinning again and adding whatever they get to their first spin or stopping after their first spin and keeping what they have. If they spin again, but go over $1.00, they automatically lose. The winner is the player with the highest total. If there is a tie, the tied players compete in a spinoff which simply has the effect of choosing one of them at random.

2. There are a lot of calculations in this problem. A computer may be helpful.

1.1.3) Prove
\[ \int_0^s \int_0^{s-x} \cdots \int_0^{s-x} dx^n = \frac{s^n}{n!} \left[ \frac{d^n}{dx^n} (\sec x + \tan x) \right]_{x=0} \]

**Note:** The dummy variables of integration can really be thought of like \( x_1, x_2, \) etc. This problem is currently unsolved.