Office HRS today CANCELLED
but see me after class for any brief questions

#18 p.76

\[ U = 100 \text{ workers} \]
\[ A = \text{college graduates} \]
\[ B = \text{union members} \]

- 60 not college grads
- 20 union members
- 30 union, college grads

How many were neither college grade nor union members?

\[ |A'| = 60 \quad B' \cap A = \text{? non union college grads} \]
\[ |B'| = 30 \quad \text{Want} \quad |A' \cap B'| = 1 \quad (A \cup B)' \]

Technique: introduce a variable or two (or three).

\[ x = |A \cap B| \]
\[ y = |B \cap A'| \]
\[ z = |(A \cup B)'| \]

**This is what we want!**

\[ |U| = |A| + |A'| \]
\[ 100 = |A| + 60 \quad \Rightarrow \quad |A| = 40 \]
\[ |A'| = 20+x \quad \Rightarrow \quad 40 = 20+x \quad \Rightarrow \quad x = 20 \]

\[ U = |B| + |B'| \quad \Rightarrow \quad 100 = 30 + |B'| \quad \Rightarrow \quad |B'| = 70 \]

\[ |B'| = 20+z \quad \Rightarrow \quad 70 = 20+z \quad \Rightarrow \quad z = 50 \]

To figure out y:

\[ |B| = x+y = 20+y \quad \Rightarrow \quad y = 10 \]
\[ n(S) = 3 \quad n(S \cup T) = 6 \quad n(T) = 4 \]
\[ n(S' \cup U \cap T') = 9 \]

R not relevant

\[ x + y = 3 \]
\[ x + y + z = 6 \]
\[ y + z = 4 \]
\[ w + x + z = 9 \]
\[ x + 2 + z = 9 \]
\[ z = 3 \]

\[ S' \cup U \cap T' = (S \cap T)' \]

Always using the principle of inclusion and exclusion.

\[ |S \cup T| = |S| + |T| - |S \cap T| \]
\[ 6 = 3 + 4 - |S \cap T| \quad \Rightarrow \quad |S \cap T| = 1 \]
\[ n(U) = 64 \quad n(R \cup S \cup U) = 45 \quad n(R) = 22 \]
\[ n(T) = 26 \quad n(R \cap S) = 4 \quad n(S \cap T) = 6 \]
\[ n(R \cap T) = 8 \quad n(R \cap S \cap T) = 1 \]

**Venn Diagram:**

\[ R \quad S \quad T \quad U \]

\[ 11 \quad 3 \quad 5 \]

\[ 7 \quad 1 \quad 5 \]

\[ 13 \]

**Working from inside out by region:**

\[ |T| = 26 \quad 7 + 1 + 5 + ? \]
\[ |R| = 22 \quad 7 + 3 + 1 + ? \]

\[ n(R \cup S \cup U) = 45 \quad 11 + 3 + 7 + 1 + 5 + 13 + ? \]
\[ = 40 + ? \]

\[ |U| = |R \cup S \cup U| + ? \]
\[ 64 = 45 + ? \]

**Using PIE**

\[ n(R \cup S \cup U) = n(R) + n(S) + n(T) - n(R \cap S) - n(R \cap T) - n(S \cap T) + n(R \cap S \cap T) \]
\[ \Rightarrow 45 = 22 + n(S) + 26 - 4 - 8 - 6 + 1 \]
\[ 45 = 31 + n(S) \Rightarrow n(S) = 14 \]

Get rest by doing more PIE.
Counting exercise: I flip a coin, then I roll a 6-sided die, how many outcomes are there possible?

Flip a coin, roll a 6-sided die, then pick one of 3 dogs, cat, fish.

How many outcomes:

\[ 2 \times 6 \times 3 = 36 \]

\[ \text{MULTIPLICATION PRINCIPLE} \]

If a task is composed of two operations, and the first operation has \( m \) outcomes, and for each of the \( m \) outcomes, there are \( n \) outcomes for the second operation, then there are \( m \cdot n \) outcomes of the task.
Does this violate the mult. principle?

Task: flip a coin. If heads, pick a card from all the red cards in a deck of 52 cards. If tails, pick a card from all the black cards in a standard deck.

General mult. principle: Suppose a task consists of operations performed consecutively. Suppose operation 1 can be performed in \( m_1 \) ways, and for each of these, operation 2 can be performed in \( m_2 \) ways, and for each of these, operation 3 can be performed in \( m_3 \) ways, etc.

Then the total \( m \) ways to perform the task is \( m_1 \cdot m_2 \cdot m_3 \cdots m_e \).

Example

I want to assign social security numbers to people working in the U.S. These are 9 digits long. How many ways to do this?

The task is a sequence of 9 operations by picking a # from 0 to 9.

Operation 1: 10 ways \( m_1 = 10 \)
2: 10 ways \( m_2 = 10 \)
3: \( m_3 = 10 \)

By mult. principle, there are \( 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^9 \) different #s.
Special Case of Mult Princ. is PERMUTATIONS

Ex. I have 4 books (Math, Sci Fi, Novel, Self Help)
How many ways can I arrange them on the shelf?

Operation 1: put 1st book on shelf \( m_1 = 4 \)
2: put 2nd book to right of 1st book \( m_2 = 3 \)
3: put 3rd book to right of 2nd book \( m_3 = 2 \)
4: put last book to right of 3rd book \( m_4 = 1 \)

Ans. by mult. prrn. is \( 4 \cdot 3 \cdot 2 \cdot 1 = 4! \)

Ex. Design a test with 20 problems in order from a test bank with 100 problems. How many ways can I write the test?

Operation i is pick the i-th problem for the test \( (i=1, 2, \ldots, 20) \).

\[
\begin{align*}
m_1 &= 100 \\
m_2 &= 99 \\
m_3 &= 98 \\
m_4 &= 97 \\
m_5 &= 96 \\
&\vdots \\
m_{20} &= 81 \\
\end{align*}
\]

Ans. \( 100 \cdot 99 \cdot 98 \cdot \ldots \cdot 81 \)