Math 105 Summer 2002—Exam 2
Instructor: J. Shapiro

Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Notes, books are NOT ALLOWED. You may not use a graphing utility.

[10] 1. Perform the division: \( \frac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1} \).

\[
\begin{array}{c}
2x^2-1) 6x^4+10x^3+13x^2-5x+2 \\
2x^2-1 \\
= 3x^2+5x+6 \\
s \hline
3x^2+5x+6 \\
6x^4+10x^3+13x^2-5x+2 \\
-6x^4-3x^2 \\
10x^2+5x \\
-10x^2-5x \\
16x+2 \\
-16x^2-2 \\
10 \\
\end{array}
\]

[8] 2. Using Descarte rule of sign, determine how many positive and how many negative real roots the polynomial \( x^4 + 3x^3 - x^2 - x - 2 \) might have.

\( f(-x) = x^4 - 3x^3 - x^2 + x - 2 \)

one positive real root

3 or 1 negative real root

[12] 3. Evaluate the expressions

(a) \( \log_3 27^5 \).

\[
27 = 3^3 \\
= \log_3 (3^3)^5 \\
= 5 \log_3 3 \\
= 15 \log_3 3 = 15 \cdot 1 = 15
\]

(b) \( \log_5 \frac{1}{25} \).

\[
5^{-2} = \frac{1}{25} \\
\sqrt{-2} \text{ (not real)}
\]
4. Let $f(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$

(a) Using the rational root theorem, list all possible rational roots of $f(x)$.

$$\pm 1, \pm 2, \pm 3, \pm 6$$

(b) Find all the roots of $f(x)$.

\[
f(x) = x^4 - 3x^3 - 3x^2 + 11x - 6
\]

\[
f(x) = (x-1)(x^3 - 2x^2 - 5x + 6)
\]

\[
x^3 - 2x^2 - 5x + 6 = 0
\]

\[
f'(x) = 3x^2 - 4x + 3
\]

\[
g(x) = x^3 - 2x^2 - 5x + 6
\]

\[
x = 1
\]

\[
x^2 - 2x - 3x + 3 = 0
\]

\[
x^2 - 4x + 3 = 0
\]

\[
(x-1)(x-3) = 0
\]

(c) Graph $f(x)$.
5. Find the intercepts and asymptotes of the function \( f(x) = \frac{x^2}{x^2 - 2x - 3} \), then graph.

**H.A.** \( y = 1 \)

**V.A** \( (x^2 - 2x - 3) = (x - 3)(x + 1) \)

\( x = 3, x = -1 \)

Intercepts at \((0, 0)\)

\[
\begin{array}{c|c}
 x & y \\
\hline
 x = 3 & + \\
 x < 3 & - \\
 x > -1 & - \\
 x < -1 & + \\
\end{array}
\]

6. Find a rational function with the following properties: vertical asymptotes \( x = 3, x = 5 \) and horizontal asymptote \( y = 4 \). Extra Credit (2pts): Make the \( x \) intercept at 1.

\[
\frac{4x^2}{(x-3)(x-5)}
\]

Extra Credit \( \frac{4x^2 - 4}{(x-3)(x-5)} \)
7. A man invests some money at 6% compounded continuously. After 10 years, he has $9,000. How much money did he invest initially? (Just set up the formula.)

\[ 9,000 = P_0 \, e^{(0.06)10} = P_0 \, e^{0.6} \]

So \[ P_0 = \frac{9,000}{e^{0.6}} \]

8. Graph the function \( f(x) = -2^x + 3 \) — reflected across \( x \)-axis.

Then shift upward by 3 units.

9. Solve for \( x \): \( \log(x + 3) = \log x + \log 3 \)

\[ \log(x+3) = \log 3x \]

So \( x + 3 = 3x \)

3 = 2x

\[ \frac{3}{2} = x \]