Math 105 Summer 2002—Exam I
Instructor: J. Shapiro

Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Notes, books and graphing calculators are NOT ALLOWED.

1. Solve for $x$ in the following:

(a) $x^2 = 3x + 10$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5, \quad x = -2$$

(b) $|3x - 4| \leq 25$

$$-25 \leq 3x - 4 \leq 25$$

$$-21 \leq 3x \leq 29$$

$$-\frac{7}{3} \leq x \leq \frac{29}{3}$$

(c) $\frac{2x}{x - 1} > 3$

$$\frac{2x}{x - 1} - 3 > 0 \Rightarrow \frac{2x - 3(x - 1)}{x - 1} > 0$$

$$- \frac{x + 3}{x - 1} > 0$$

$$x = 3, \quad x = 1$$
2. Determine the domain of the function \( f(x) = \frac{3}{\sqrt{3 - 2x}} \)

\[
\begin{align*}
3 - 2x &> 0 \\
3 &> 2x \\
x &< \frac{3}{2} \\
\Rightarrow & \quad (-\infty, \frac{3}{2})
\end{align*}
\]

3. For the function \( f \) given by the graph below find the following:

(a) \( f(2) \). \( f(2) = 0 \)

(b) The intervals where \( f \) is increasing. \((-\infty, 0), \left(\frac{3}{2}, \infty\right)\)

(c) The value(s) of \( x \) where \( f(x) = 0 \). \( x = -1, 1, 2 \)

(d) The intervals where \( f \) is negative. \((-\infty, -1), (1, 2)\)
4. Graph the function \( f(x) = \begin{cases} 3 - x & \text{if } x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases} \).

5. A farmer has 1200 feet of fencing. He is going to fence off a rectangular field that borders a straight river. He is also going to divide the field into two pieces by putting in a fence perpendicular to the river. Assuming he needs no fencing along the river, determine the dimensions of this field that will maximize the enclosed area. (Hint: Draw a picture.)
6. Write the quadratic function \( f(x) = -x^2 - 4x + 5 \) in standard form, then determine the range of the function.

\[
-x^2 - 4x + 5 = -(x^2 + 4x + 4) + 5 + 4 = -(x + 2)^2 + 9
\]

\( V = (-2, 9) \) - concave down

Range: \((-\infty, 9]\)

7. Let \( f(x) = x^3 + 2 \) and \( g(x) = \frac{1}{2x - 1} \). Find \( g \circ f(x) \).

\[
g(f(x^3 + 2)) = \frac{1}{2(x^3 + 2) - 1} = \frac{1}{2x^3 + 3}
\]

8. Use the graph of \( f \) below to graph the function \( g(x) = -f(x - 2) + 2 \).