Project 2
Fixed Point Iteration
and Convergence

The equation

\[ 2x^3 + 3x + 5 \cos e^{\sin x} = 1 + x^2 \]

has three solutions. The goal of this project is to use Fixed Point Iteration to find all three roots and analyze the linear convergence rate of FPI to the roots.

1. Use the Fixed Point Iteration method to calculate all three roots, each rounded to 10 correct decimal places. Each root \( r \) will be a fixed point of FPI with a particular \( g(x) \). You may find it necessary to use more than one \( g(x) \) to find them all, and you may need to vary the initial guesses as well. For each root of the equation, report the fixed point rounded accurately to 10 decimal places, the \( g(x) \) and initial guess you used, and the number of FPI steps required to reach this accuracy. Note: You may NOT use Newton’s Method \( g(x) \).

2. For each fixed point \( r \), use calculus to determine \( S = |g'(r)| \).

3. For each fixed point \( r \), use your Matlab calculations to approximate the convergence rate

\[ \lim_{i \to \infty} \frac{e_{i+1}}{e_i} \]

of the Fixed Point Iteration. Our theory says that this limiting ratio should be \( S \). Show that your approximate limits match with the answers in part 2.

Begin your report by answering the three questions above. Print out the Matlab code used and your Matlab session, and include these with your report.

Due: Thurs., June 18