MATH 213 – 17 FEBRUARY 2003 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given.

1. (2 pts. each) Identify whether each of the following quantities is a vector or scalar quantity. In each problem below, \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \) denote 3-dimensional vectors.

   (a) \( \mathbf{a} \cdot \mathbf{b} \), (b) \( \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \), (c) \( \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a} \), (d) \( \mathbf{a} \times \mathbf{b} \), (e) \( |\mathbf{a} \times \mathbf{b}| \), (f) \( \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) \)

2. (10 pts.) A woman walks due east at 5 miles per hour on the deck of a ship that is moving northeast at 18 miles per hour. Find a vector giving the velocity of the woman relative to the surface of the water (this vector can be in whatever form you like). What is the woman’s speed relative to the surface of the water?

3. (6 pts. each) Let \( \mathbf{a} = 6\mathbf{i} - 5\mathbf{j} + \mathbf{k} \) and \( \mathbf{b} = \mathbf{i} + \mathbf{k} \).

   (a) Compute \( \mathbf{b} - \mathbf{a} \) and \( \mathbf{a} - 6\mathbf{b} \).

   (b) Compute \( \mathbf{a} \cdot \mathbf{b} \).

   (c) Find \( |\mathbf{a}|, |\mathbf{b}| \).

   (d) Find the angle between the vectors \( \mathbf{a} \) and \( \mathbf{b} \) correct to the nearest degree.

   (e) Find \( \text{proj}_\mathbf{a}(\mathbf{b}) \).

   (f) Find \( \text{comp}_\mathbf{a}(\mathbf{b}) \).

4. (12 pts. each)

   (a) Find an equation for the plane containing the points \( A = (2, -1, 5), B = (4, 4, 1) \) and \( C = (2, 1, -3) \).

   (b) Find parametric and symmetric equations for the line that contains the point \( B \) and is perpendicular to the plane you found in part (a).

   (c) Find the distance from the point \( C \) to the line you found in part (b). (Hint: You may do this problem any way you like, but feel free to use the formula \( d = |\mathbf{a} \times \mathbf{b}| / |\mathbf{a}| \) where \( \mathbf{a} \) is the direction vector of the line and \( \mathbf{b} = \overrightarrow{PC} \) where \( P \) is any point on the line.)

5. (10 pts.) Find symmetric and parametric equations for the line of intersection of the planes given by \( x - 2y + 3z = 1 \) and \( x + y + z = 2 \).