Answer all of the following questions on the answer sheets provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (8 pts. each) The table below gives values of three functions.

<table>
<thead>
<tr>
<th>t</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(t)</td>
<td>15</td>
<td>22</td>
<td>28</td>
<td>33</td>
</tr>
<tr>
<td>G(t)</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>H(t)</td>
<td>15</td>
<td>17</td>
<td>20</td>
<td>24</td>
</tr>
</tbody>
</table>

(a) Which of the functions in the above table is linear, which is concave up and which is concave down? Explain your answer.

(b) Find an equation for the linear function in the above table.

2. (8 pts. each) The population, $P$, in millions, of Nicaragua $t$ years from the beginning of 1990 is given by $P(t) = 3.6(1.034)^t$.

(a) Write $P(t)$ in the form $P(t) = P_0 e^{kt}$.

(b) Find the average rate of change of the population between 1990 and 2000. Give your answer in correct units.

(c) Find the instantaneous rate of change of the population at the beginning of 1997. Give your answer in correct units. (Hint: Recall that one interpretation of the first derivative is as the instantaneous rate of change.)

3. (6 pts. each) A flu epidemic has broken out in a certain town. Let $s(t)$ denote the number of people who currently have the flu $t$ weeks after the epidemic breaks out. Make a rough sketch of the graph of the function $s(t)$ given each of the following mathematical statements. Indicate whether each statement is good news or bad news for the town.

(a) $s'(t) > 0$ and $s''(t) > 0$.

(b) $s'(t) < 0$ and $s''(t) > 0$.

(c) $s'(t) > 0$ and $s''(t) < 0$.

(d) $s'(t) < 0$ and $s''(t) < 0$.

4. (8 pts. each) The following table gives the cost $C(q)$ and revenue $R(q)$ functions for a certain good.

<table>
<thead>
<tr>
<th>q</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(q)</td>
<td>0</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>47</td>
<td>51</td>
<td>55</td>
</tr>
<tr>
<td>C(q)</td>
<td>5</td>
<td>13</td>
<td>19</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>29</td>
<td>35</td>
<td>44</td>
<td>53</td>
<td>65</td>
</tr>
</tbody>
</table>

(a) Use the data in the table above to compute the profit function $P(q) = R(q) - C(q)$ for $q = 0$ through $q = 10$. For which value of $q$ is profit maximized?

(b) Use the data in the table to estimate the marginal revenue $R'(q)$ and marginal cost $C'(q)$ for the value of $q$ you found in part (a). What do you observe about marginal revenue and marginal cost at this $q$?
5. (10 pts. each) Find the first derivative of the following functions.

(a) \(4\sqrt{x} + \frac{4}{\sqrt{x}}\)
(b) \(25x^2 e^{-x}\)
(c) \(\frac{z}{1 + z^2}\)

6. (12 pts. each) Find the first and second derivatives of the following functions.

(a) \(f(x) = 6x^3 + 4x^2 - 2x\)
(b) \(g(t) = \cos(3t^2)\)
(c) \(h(z) = \ln(5z^2 - 1)\)

7. (10 pts. each) Let \(f(x) = x^4 - 8x^2 + 5\).

(a) Find all critical points of \(f(x)\).
(b) Find the intervals on which \(f(x)\) is increasing and decreasing.
(c) Find all \(x\) for which \(f(x)\) has local maxima and minima.
(d) Find the intervals on which \(f(x)\) is concave up and concave down.
(e) For \(-1 \leq x \leq 3\), find the values of \(x\) for which \(f(x)\) has its global maximum and minimum.

8. The rate of change of the world’s population in millions of people per year is given in the following table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Change</td>
<td>37</td>
<td>41</td>
<td>77</td>
<td>78</td>
<td>86</td>
</tr>
</tbody>
</table>

(a) (8 pts.) Assuming that the rate of population growth is steadily increasing between 1950 and 1990, find upper and lower estimates of the total change in population between 1950 and 1990.
(b) (6 pts.) If the population in 1950 was 2500 million people, what is your best estimate of the world population in 1990?

9. The following table gives values of the function \(f(t)\) at different values of \(t\). Assume that \(f(t)\) is decreasing throughout the interval \(10 \leq t \leq 26\).

<table>
<thead>
<tr>
<th>(t)</th>
<th>10</th>
<th>14</th>
<th>18</th>
<th>22</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(t))</td>
<td>50</td>
<td>48</td>
<td>44</td>
<td>36</td>
<td>24</td>
</tr>
</tbody>
</table>

(a) (8 pts.) Find upper and lower estimates for the definite integral \(\int_{10}^{26} f(t) \, dt\).
(b) (6 pts.) How close would the upper and lower estimates be if values of \(f(t)\) were given every \(\Delta t = 1\) instead of every \(\Delta t = 4\)?