Math 214 Answers Exam II

1. Let $x(t)$ be the solution of the initial valued problem: $x'' - 4x = 0, x(0) = 1, x'(0) = 2$. Find $x(1)$.
   $e^2$

2. Let $x(t)$ be the solution of the initial valued problem: $x'' + x = 0, x(0) = 1, x'(0) = 1$. Find $x(1)$.
   $\sin 1 + \cos 1$

3. Find the general solution of the differential equation $x''(t) - 4x'(t) + 4x(t) = 0$.
   $C_1e^{2t} + C_2te^{2t}$

4. Find the general solution of the differential equation $x''(t) + 4x'(t) + 5x(t) = 0$.
   $e^{-2t}(C_1 \sin t + C_2 \cos t)$

5. Given the fact that $\sin t$ is a (particular) solution of $x'' - 3x' + 2x = \sin t - 3 \cos t$, find the general solution of the equation.
   $C_1e^t + C_2e^{2t} + \sin t$

6. The differential equation $x'' - 3x' + 2x = 2e^{3t}$ has a (particular) solution of the form $Ae^{3t}$

7. The differential equation $x'' - 3x' + 2x = 2e^{2t}$ has a (particular) solution of the form $Ae^{2t}$

8. Write $\cos t - \sin t$ in the form $A \cos(\omega t - \phi)$.
   $\sqrt{2} \cos(t + \frac{\pi}{4})$

9. In an experiment, a 5-kg mass is suspended from a spring. The displacement of the spring-mass equilibrium from the spring equilibrium is measured to be 0.49 m. The spring mass system is suspended in a viscous solution that dampens the motions according to $R(v) = -0.125v$. Assume that the spring is initially displaced 0.25 m downward from the spring-mass equilibrium and then given a sharp downward tap, imparting an instantaneous downward velocity of 0.45 m/s. Set up the initial value problem that models this experiment.
   $x'' + 0.025x' + 20x = 0, x(0) = 0.25, x'(0) = 0.45$
10. Let \( f(t, x) \) be a continuous function for \(-\infty < t < \infty, -\infty < x < \infty\). Then the IVP \( x' = f(t, x), x(0) = 1 \) has a solution defined on \(-\infty < t < \infty\).

**False**

11. Let \( f(t, x) \) be a continuous function for \(-\infty < t < \infty, -\infty < x < \infty\).
Then the IVP \( x' = f(t, x), x(0) = 1 \) has a solution \( x(t) \) defined on an open interval containing 0.

**True**

12. Determine the equilibrium solutions of the differential equations \( x' = x(1 - \frac{x}{2}) \).

\( x = 0, x = 2 \)

13. (optional) Let \( y_1 \) be a known solution of the (DE) \( y'' + p(t)y' + q(t)y = 0 \). Then for \( vy_1 \) to be also a solution of (DE), \( v \) must satisfy

\( v'' + (2\frac{y_1'}{y_1} + p(t))v' = 0 \)