Answers to new problems 1.1 - 7.2.

1. (b) False. (d) True. (e) False. (f) False. (i) False.

2. (b) True.

3. (b) \( x \) is any number complex or real.
   (d) There exists an integer not divisible by a prime.
   (f) \((a, b, c) = a(bc)\) for all \(a, b, c\).
   (g) There exists \(x > 0\) and some \(y\) such that \(x^{-y} < 0\).
   (i) For any infinite set, some proper subset is not finite.

4. (b) Converse: \(x \neq 1 \rightarrow x^2 = 1\)
   Contrapositive: \(x \neq 1\) and \(x + 1 \rightarrow x^2 \neq 0\).
   (c) Conv.: \(a = 0\) or \(b = 0\) \(\rightarrow\) \(ab = 0\)
   Contra.: \(a\neq 0\) and \(b\neq 0\) \(\rightarrow\) \(ab \neq 0\).
   (f) Conv.: \(\text{if } a = b + c\), then \(\text{ABC is a right tri.}\)
   Contra.: \(\text{if } a \neq b + c, \text{ then } \text{ABC is not a right tri.}\)

5. (b) \(x \geq 0\) for all real nos. \(x\).
   (e) For every set of primes \(p_1, \ldots, p_n\), there exists a prime not in this set.
   (i) For every real no. \(x \geq 0\), there exists a real no. \(a\) such that \(a^2 = x\).

6. Not possible.

2:1

1. (b) \( \{1, 3, 5, 15, -1, -3, -5, -15\} \)
   (c) \( \{1, -1, 0, 1, 2, 3\} \)
   (e) \(\emptyset\)

3. (b) \(\emptyset, \{1, 1\}, \{1, 2\}\)
   (c) \(\emptyset, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\)
   (d) \(\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\)
   (e) \(\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\)
   (f) \(\emptyset, \{1, 2\}\)

5. (b) False.
   (c) True.
   (e) False.
   (f) False.
   (g) False.
2.1. 6. (b) \{1,2,3,4,5\}  (c) \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}

9. (a) \{a,b,c,d\}
   (ii) \{a,b, c, d\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}
   (iv) \{a, b\}, \{b, c\}, \{c, d\}
   v. \emptyset

(b) 16

2.2. 1. (b) \text{AUC} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}
   \text{BNC} = \{0, 2\}
   \text{B \setminus C} = \{-1, 1, 3, 4, 5\},
   \text{A \setminus B} = \{-1, 0, 6\},
   \text{C \times (B \setminus C)} = \{(0, -1), (0, 1), (0, 3), (0, 4), (0, 5)\}
   \text{(A \times (B \setminus C)) =} \{(0, -1), (0, 1), (0, 3), (0, 4), (0, 5)\}
   \text{A \times (B \setminus C)} = \{2, 6\}
   \text{(B \cup \emptyset)} \cap \emptyset = \emptyset

(c) \text{S} = \{(1, -1), (2, 0), (3, 1), (4, 2), (5, 3), (6, 4)\},
   \text{T} = \{(1, 2), (2, 2)\}

2. (c) \text{Z \cap \{S \cup T\}} = \{2, 5, 25, 56\} = (\text{Z \cap S}) \cup (\text{Z \cap T}). \text{ The two sets are equal}
   (d) \text{Z \cup (S \cap T)} = \{1, 2, 3, 4, 5, 6\} = \{(\text{Z \cup S}) \cap (\text{Z \cup T})\}, \text{ The two sets are equal}

7. (b) \{4, 6, 8\}
   (c) \{10, 1\}

6. (b) \text{A}^c = (-\infty, -3] \cup [4, \infty)
   (c) \text{A}^c = \text{IR}

7. (b) 2^{n-2}
   (c) 4

9. (c) m \not\in \text{P}
   (d) \text{CS} \cap \text{T} \in \text{P}
   (e) (\text{MU CS}) \cap \text{P} \subseteq \text{T}^c

16. 16. (1,1) \in \text{A} \Rightarrow (2, 1), (2, 2) \in \text{A}
      (2,1) \in \text{A} \Rightarrow (3, 1), (3, 2) \in \text{A}
      (2,2) \in \text{A} \Rightarrow (3, 3) \in \text{A}
      (3,1) \in \text{A} \Rightarrow (4, 1), (4, 2) \in \text{A}
      (3,2) \in \text{A} \Rightarrow (4, 2) \in \text{A}
      (3,3) \in \text{A} \Rightarrow (4, 4) \in \text{A}

   \begin{align*}
   \text{plot this points and you see that it is plausible.}
   \end{align*}
2.2. 25. \( b \neq B = C \).

For example, \( A = \{1, 2\}, B = \{1, 2, 3\}, C = \{1, 2, 3\} \).

Then \( A \gcd B = A \gcd C = A \) but \( B \neq C \).

2.3. 3. (b) (in most cases) reflexive, (in somewhat fewer cases) symmetric, certainly not transitive.

(c) reflexive, symmetric, transitive

(d) not reflex, not symmetric, not transitive.

5. (b) \( \{1, 1, 2, 2, 3, 3, 1, 2, 2, 3, 3\} \)

(c) \( \{1, 2, 2, 3, 3, 1, 1, 2, 1, 3, 2\} \).

(d) \( \{1, 2, 1, 3, 2, 3\} \).

(e) \( \{1, 1, 2, 2, 3, 3, 1, 2, 1, 2, 1, 3, 2\} \).

(f) \( \{1, 1, 2, 2, 3, 3, 1, 2, 1, 2, 1, 3, 2\} \).

(g) \( \{1, 1, 2, 2, 3, 3, 1, 2, 1, 2, 1, 3, 2\} \).

6. Yes.

9. (a) not reflexive, not symmetric, antisymmetric, transitive.

(b) reflexive, symmetric, not antisymmetric, transitive.

(c) reflex, not symmetric, not antisymmetric, not transitive.

(d) reflex, symmetric, not antisymmetric, not transitive.

(e) reflex, not symmetric, antisymmetric, not transitive.

5.1. 1. (c) 117

(e) 59

(f) 0 if \( n \) is odd; 1 if \( n \) is even.

3. Did in class.

4. (b) \( a_1 = 5, a_n = a_{n-2} \) \( \forall n \geq 3 \).

(c) \( a_1 = 1, a_2 = 2, a_n = a_{n-1} + a_{n-2} \) \( \forall n \geq 3 \).

(d) Two possibilities:

\( a_n = a_{n-1} + (-1)^{n+1} \).

Also \( a_1 = 1, a_2 = a_1 + (-1)^2 \) \( \forall n \geq 3 \).

2. (b) 17, 8, 4, 2, 2, 1, 1

(c) 18, 9, 4, 2, 2, 1

(d) 100, 50, 25, 12, 6, 3, 1
5. Write the first seven terms: \(0, \frac{1}{4}, \frac{2}{8}, \frac{3}{16}, \frac{4}{32}, \frac{5}{64}, \frac{6}{128}\).

\[ a_n = \frac{1}{3} \left(1 - \left(-\frac{1}{2}\right)^n\right). \]

13. Use strong form of math induction.

(f) 1023 does not belong to the sequence.

14. 127 is the 1376th term.

26. (a) \(2^{4^n} = 2^{10} \approx 2^{14}\)

(b) \(\approx \frac{2^{14+4}}{1-2} = 2^{18}\)

32. \(210 - 25 = 765.6\)

5.3

2. \(a_n = \frac{1}{8} \left[(-7)^{2n^2} + 207\right]\)

9. \(a_n = 7(2^{n-1}) + 3(5^{n-1})\)

6. \(a_n = 7 - 2(-6)^n\)

8. \(a_n = 40 - 3n\)

6.1

3. M: all students, J: those known JAVA, H: those known HTML.

(a) N \(\subseteq\) M

(b) 10

(c) 7

(d) 17

(e) 8

9. (a) 1

(b) 34

(c) 351

(d) 29

10. 109

19. \(A \cap \emptyset \neq \emptyset\).

6.2

2. a. 10
   b. 40

6. a. 17576
   b. 456976
   c. 950508
   d. 2610

19. (a) 36
    (b) 7776
    (c) 6^n
    (d) 6^n - 6
6.1  22. (a) 15600000
    (b) 11232000

23. (a) 380
    (b) 570
    (c) 119
    (d) 839

6.3  7. (a) 18
    (b) 14

7.1  2. 336
4. \( P(30,8) = 30.23 \ldots \)
6. 43200
7. (a) 141
    (c) 46111
    (d) 94101
14. (b) 96
    (c) 480

7.2  4. 35
7. (a) 13983616
    (c) 6096454
15. (a) 84
    (b) 49
    (c) 10
16. (a) 21324
    (b) 14704
    (c) 25194
    (d) 8085
19. (b) 120
    (c) 36