Exam 3
Math 125
December 2, 2004
Lim

1. How many ways can you select 7 books from 35 books?
\[ \binom{35}{7} = \frac{35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29}{7!} = 6724520 \]

2. A student must answer exactly 7 questions out of 10 on a final exam. In how many ways can she choose the questions to answer if she must answer at least 3 of the last 5 questions?
\[ \binom{5}{4} \times \binom{5}{3} + \binom{5}{3} \times \binom{5}{2} + \binom{5}{2} \times \binom{5}{1} = 50 + 50 + 10 = 110 \]

3. In how many ways can 4 boys and 6 girls line up so no two boys stand next to each other?
\[ 6! \times P(7, 3) = 720 \times 7 \times 6 = 60480 \]

4. In how many ways can 9 people sit around a circular table if 2 of the people must be separated?
\[ 8! - 2 \times 7! = 30240 \]
Or, \( 6! \times P(7, 2) = 6! \times 7 \times 6 = 30240 \).

5. In how many ways can the letters of the word REARRANGED be rearranged?
\[ \frac{10!}{3!2!2!} = 151200 \]

6. There are 15 varieties of chocolates available and Linda wants to buy 7 chocolates. How many choices does she have?
\[ n = 15, r = 7, \binom{n+r-1}{r} = \binom{22}{7} \]

7. In how many ways can 6 marbles of different colors be put in 5 distinct boxes?
\[ n = 5, r = 6, n^r = 5^6 \]

8. How many nonnegative integer solutions are there to the equation \( x + y + z + w = 65 \)?
\[ n = 4, r = 65, n + r - 1 = 68, n - 1 = 3, \binom{n+r-1}{r} = \binom{68}{3} \]
9. (continued) subject to $x \geq 3, y \geq 4, z \geq 5$?

$$n = 4, r = 65 - 3 - 4 - 5 = 53, n + r - 1 = 56, \binom{n+r-1}{r} = \binom{56}{3}$$

10. 40 students take an exam. For the purposes of grading, the teacher asks the students to exchange papers so that no one marks his or her own paper. In how many ways can this be accomplished?

$$40! * (1 - 1 + 1/2! - 1/3! + \ldots + 1/40!) = 40! * B(40)$$

11. Find the coefficient of $a^6b^9$ in the binomial expansion of $(a + b)^{15}$.

$$\binom{15}{6}$$

12. Find the sum $\binom{45}{0} + \binom{45}{1} + \binom{45}{2} + \ldots + \binom{45}{45}$.

$$(1 + 1)^{45} = 2^{45}$$

13. Find the sum $\binom{45}{0} - \binom{45}{1} + \binom{45}{2} - \ldots - \binom{45}{45}$.

$$(1 - 1)^{45} = 0$$

14. The subgraphs used in the game of Instant Insanity must satisfy: [i]each graph has 4 vertices; [ii] the degree at each vertex is 2; [iii] each graph has 4 edges, one from each cube. The subgraphs need not be connected.

15. (continued) Given the two subgraphs in Fig. 1, a solution to the game is:

cube 4: BGBR, cube 3: WRGG, cube 2: RBRW, cube 1: GWWB

16. Determine whether the graph in Fig. 2 is bipartite.

It is bipartite b/c it is 2-colorable.