Suppose $U$ has orthonormal columns

$$W = \text{col}(U) = \text{row}(U^T); \quad W^\perp = \text{nul}(U^T)$$

$$\text{nul}(U) = \mathbb{E}_{0^2}; \quad U^T U \vec{x} = \vec{x}; \quad U U^T \vec{y} = \vec{y}$$

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**Figure 4.2** The true action of $A$ times $x$: row space to column space, nullspace to zero.