Clearly show the calculations to solve the following. Except for the first problem, it is not necessary to actually calculate the answer. Follow the Honor Code.

1. How many ways can you choose 99 people from a group of 100?
\[ C(100, 99) = C(100, 1) = 100 \]

2. From a group of 500 people, 350 play tennis, 250 play hockey, 200 play both. How many a) play tennis, but not hockey; b) play neither?
\[ m(U) = 500 \]
\[ m(T) = 350 \]
\[ m(H) = 250 \]
\[ m(T \cap H) = 200 \]
\[ a) m(T \setminus H') = m(T) - m(T \cap H) = 350 - 200 = 150 \]
\[ b) m(H \cup T) = m(H) + m(T) - m(H \cap T) = 250 + 350 - 200 = 400 \]
\[ m(H' \cap T') = m(U) - m(H \cup T) = 500 - 400 = 100 \]

3. If you have 6 sandwiches, 3 soups, and 4 salads to choose from, in how many ways can you choose a sandwich and either a soup or a salad, but not both?
\[ 6 \cdot (3 + 4) = 42 \]

4. How many 12 letter words are there:
   a) altogether;
   b) that have 4 R's, 3 S's, and 5 T's;
   c) at most one Z;
   d) at least one Z?
\[ a) \text{ choose 12 from 26; rep YES; order YES} \]
\[ b) \text{ partition 12 into parts of 4, 3, and 5} \]
\[ C(12,4) \cdot C(8,3) \cdot C(5, 5) = \frac{12! \cdot 8! \cdot 5!}{4! \cdot 3! \cdot 5!} \]
\[ c) \text{ no Z's: } 25^{12} \]
\[ \text{ only Z: } 12 \cdot 25^{11} \quad \text{so} \quad 25^{12} + 12 \cdot 25^{11} \]
\[ d) \text{ at least one is complement of none} \]
\[ 26^{12} - 25^{12} \]
5. A population contains 8 men and 12 women. In how many ways can you:
   a) choose a sample of 3 men and 6 women;
   b) divide them into 4 groups of 5 each?

   a) choose 3 men in \( C(8,3) \) ways
   b) choose 6 women in \( C(12,6) \) ways

   so \( C(8,3) \cdot C(12,6) = \frac{8 \cdot 7 \cdot 6 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{3! \cdot 6!} \)

b) unordered partition of 20 into parts of size 5, 5, 5, 5

\[
\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 1}{5! \cdot 5! \cdot 5!} \]

6. A salad bar offers 8 ingredients and 4 dressings. How many different salads that contain at most 1 dressing are possible?

   a) each ingredient in \( 2^8 \) ways
   b) choose dressing in \( 1+4 = 5 \) ways

   so \( 2^8 \cdot 5 \)

7. In how many ways can 4 men and 4 women be seated:
   a) in a row; b) in a row with nobody next to someone of the same sex?

   a) choose 8 from 8; no NO; order YES
   so \( P(8,8) = 8! \)

   b) choose 1 of 8; then 1 of 4 of opposite sex; then 1 of 3 of first sex, etc.
   so \( 8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \)

8. For the sets shown in this diagram, how many are:
   a) in R and in T, but not in M; b) in exactly 1 of R,T,M;
   c) at most 1 of R,T,M?

   a) \( |5| \)
   b) \( 25 + 20 + 0 = 45 \)
   c) \( 25 + 20 + 0 + 10 = 55 \)