1. Sketch the graphs of the following. (Show your work).
   a) \( y = 12 + 6x \);
   b) \( y = 20x - 2x^2 \).

   \( y = 12 + 6x \)
   \( x = 0 \rightarrow y = 12 \)
   \( x = 1 \rightarrow y = 18 \)
   \( x = 2 \rightarrow y = 24 \)
   \( x = 3 \rightarrow y = 30 \)
   \( x = 4 \rightarrow y = 36 \)
   \( x = 5 \rightarrow y = 42 \)
   \( x = 6 \rightarrow y = 48 \)

   \( y = 20x - 2x^2 \)
   Vertex \( x = \frac{-b}{2a} = \frac{-20}{-4} = 5 \)
   \( y = 100 - 50 = 50 \)

2. Find the derivatives, if they exist, of the functions above at \( x = 8 \).
   a) \( y' = 6 \)
   \( y'(8) = 6 \)
   b) \( y' = 20 - 4x \)
   \( y'(8) = 20 - 32 = -12 \)

3. Find the equations of the tangent lines for the functions above at the points where \( x = 8 \).
   a) \( y(8) = 12 + 48 = 60 \)
   \( y = 6x + 6 \)
   \( 60 = 6 \cdot 8 + b \)
   \( 12 = b \)
   \( y = 6x + 12 \)

   b) \( y(8) = 160 - 128 = 32 \)
   \( y = -12x + b \)
   \( 32 = -12 \cdot 8 + b \)
   \( 128 = b \)
   \( y = -12x + 128 \)
4. x cases of widgets can be sold for a price of \( p = 20 - 2x \) $ per case. It costs $12 for rent and $6 per case for materials. Find:

a) The revenue that results from producing 8 cases of widgets;

b) The rate at which revenue is changing with respect to production when production is 8 cases;

c) The production that produces the maximum revenue;

d) The total cost of producing 8 cases of widgets;

e) The break even productions (zero profit.)

\[ R = p \cdot x = (20 - 2x) \cdot x = 20x - 2x^2 \]
\[ R'(8) = -12 \]

As seen in graph 1b, \( R(x) \) is max when \( x = 5 \)

\[ C = 12 + 6x \]
\[ C(8) = 12 + 48 = 60 \]

\[ P = R - C = 20x - 2x^2 - 12 - 6x = -2(x^2 - 7x + 6) = -2(x-1)(x-6) \]
\[ \text{So } x = 1 \text{ or 6} \]

These are the points where the graph in 1a and 1b intersect.

5. \( f(x) = 3x - 1 \) for \( x < 2 \), \( f(x) = x^2 + 1 \) for \( x \geq 2 \). Find, if they exist:

a) \( f(2) \);  
b) \( \lim_{x \to 2} f(x) \);  
c) \( \lim \frac{f(2+h) - f(2)}{h} \);  
d) \( f'(2) \).

\[ a) f(2) = 2^2 + 1 = \boxed{5} \]
\[ b) \lim_{x \to 2} f(x) = 3 \cdot 2 - 1 = 5 \]
\[ c) \lim_{h \to 0} \frac{(2+h)^2 + 1 - (5)}{h} = \lim_{h \to 0} \frac{(4 + 4h + h^2) + 1 - (5)}{h} = \lim_{h \to 0} \frac{4h + h^2}{h} = \lim_{h \to 0} (4 + h) = 4 \]
\[ d) \lim_{h \to 0} \frac{3(2+h)-1-5}{h} = \lim_{h \to 0} \frac{3h}{h} = 3 + 4 = 7 \]

The above is the definition of \( f'(2) \), so \( \boxed{DNE} \)

6. \( f(x) = \frac{x^2 - 1}{x^2 - x} \). Find, either finite or infinite:

a) \( \lim_{x \to 2} f(x) \);  
b) \( \lim_{x \to 0} f(x) \);  
c) \( \lim_{x \to 1} f(x) \);  
d) \( \lim_{x \to \infty} f(x) \);  
e) \( f'(2) \).

\[ a) f(2) = \frac{2^2 - 1}{2^2 - 2} = \boxed{3} \]
\[ b) \lim_{x \to 1} \frac{x + 1}{x(x-1)} = \lim_{x \to 1} \frac{x + 1}{x} = 1 \]
\[ c) \lim_{x \to 0} \frac{x + 1}{x} = -\infty \text{ since } x + 1 > 0 \text{ and } x < 0; \]
\[ \lim_{x \to 0^+} \frac{x + 1}{x} = +\infty \text{ since } x + 1 > 0 \text{ and } x > 0. \]
\[ d) f''(x) = \frac{(x^2 - x)(2x) - (x^2 - 1)(2x - 1)}{(x^2 - x)^2}; f'(2) = \frac{2 \cdot 4 - 3 \cdot 3}{4} = \boxed{\frac{-1}{4}} \]