Math 106
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Give clear explanations. Follow the Honor Code.

1. In how many ways can you choose a president, vice president, and secretary from a group of 12 people?
   - Choose 3 from 12; yes, no: yes, YES
   - 12, 11, 10

2. How many 3 letter codes are there? There are 26 letters.
   - Choose 3 from 26; yes, NO, yes, YES
   - 26^3

3. From 5 hats, 4 shirts, and 3 sweaters, in how many ways can you choose a hat and either a shirt or sweater?
   - Either shirt or sweater: 4 + 3 = 7
   - Choose one of the 7 and a hat 5 choose 1 = 5

4. A population contains 8 women and 6 men.
   a) If you choose one person, are the outcomes “woman” and “man” equally likely? Explain.
      - No, because there are more women. Odds of woman is 8 of 14: 8/14
   b) Does “all are women” imply “at least one is a woman”? Explain. YES
   c) Does “at least one is a woman” imply “all are women”? Explain. NO

   Assume you choose a random sample of 3 from the population.
   This could be 1 woman and 2 men, or 3 women, or 3 men, but not all women.

   Find the probabilities that:
   d) all are women; e) at least one is a woman.
   S: choose 3 from 14; n(S) = 14, 13, 12
   a) n(E) = 8, 7, 6
      - n(E) = 8 choose 3 = 8.2.6
      - n(E) = 7 choose 3 = 7.5.4
      - n(E) = 6 choose 3 = 6.5.4
      - n(E) = 1 - 6/7

5. You roll 2 dice.
   a) Are “both are 4” and “exactly one is a 4” equally likely? Explain.

   Find the probabilities that: a) both are 4; b) at least one is a 4; c) exactly one is a 4.
   a) Can get both 4 in only one way (4,4)
   b) Can get exactly one is a 4 in two ways: (4,1), (1,4)
      - 1/36 + 1/36 = 2/36
      - 1/18
   c) (at least one is 4) = P(4,4) + P(4,1) + P(1,4)
      - 11/36
   c) (at least one is 4) = P(4,4) + P(4,1) + P(1,4)
      - 11/36

   Find the probabilities that:
   a) both are 4
      - 1/36
   b) at least one is a 4
      - 17/36
   c) exactly one is a 4
      - 11/36
6. From a group who take a polygraph test, 95 liars fail and 5 liars pass the test; 50 truth tellers fail and 850 truth tellers pass. Find the probabilities that a person:
a) fails, given that he is a liar; b) is a liar, given that he fails;
c) is a liar; d) is a liar who fails; e) is a liar or fails.

\[
P(L) = \frac{95}{500} = 0.19
\]
\[
P(L|U) = \frac{95}{95+850} = \frac{95}{945}
\]
\[
P(U|L) = \frac{95}{500},
P(U) = \frac{850}{500},
P(L\cap U) = P(L)P(U|L) = \frac{95}{500} \times \frac{850}{500} = \frac{95 \times 850}{500^2}
\]

7. In the previous problem, are the events "is a liar" and "fails the test":
a) disjoint (mutually exclusive)? Explain.  b) independent? Explain.

a) No, some liars fail the test, so intersection is not empty.

b) No:

\[
P(L|U) = \frac{95}{945} > \frac{95}{950} = \frac{P(U|L)}{P(U)}
\]

so failing the test increases the chance that

8. The expected value of a game is -$0.35.

Does that mean that:
a) You expect to lose $0.35 every time you play the game.
b) You will lose an average of $0.35 per play;
c) you will lose 35% of your plays.
d) You will lose on most of your plays.
e) You will lose $0.35 on most your plays.