Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work. Each question is worth 5 points.

1. Find the gradient vector field of \( f(x, y, z) = xy \cos z \)

\[
\nabla f = f_x \mathbf{i} + f_y \mathbf{j} + f_z \mathbf{k} = y \cos z \mathbf{i} + x \cos z \mathbf{j} - yx \sin z \mathbf{k}
\]

2. Determine whether or not \( F(x, y) = 6xy + (3x^2 + 2y)j \) is a conservative vector field. If it is, find a function \( f \) such that \( F = \nabla f \).

\[
M = 6xy, \quad N = 3x^2 + 2y
\]

\[
M_y = 6x, \quad N_x = 6x \quad \text{Conservative}
\]

\[
\begin{align*}
f_x &= 6xy \\
f_y &= 3x^2 + 2y \\
g(x) &= y^2 \\
g'(y) &= 2y \\
\therefore f &= 3x^2y + y^2
\end{align*}
\]

3. Set up and simplify (but do not evaluate) \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F} = xy \mathbf{i} + (x + y)j \) and \( C \) is the curve given by \( y = x^2 \) from \((-1, 1)\) to \((2, 4)\).

\[
\begin{align*}
x &= t, \quad y = t^2 & \text{for } & \mathbf{r}(t) &= t \mathbf{i} + t^2 \mathbf{j} \\
\mathbf{F}(t) &= t^2 \mathbf{i} + (t + t^2) \mathbf{j} & \mathbf{r}'(t) &= 1 \mathbf{i} + 2t \mathbf{j} \\
\int_{-1}^{2} (3t^4 + 2t^2) dt &= \int_{-1}^{2} (3t^4 + 2t^2) dt
\end{align*}
\]

4. Use Green’s theorem to evaluate \( \int_C (x + y^2)dx + (1 + x^2)dy \); \( C \) the closed curve determined by \( y = x^3 \) and \( y = x^2 \) with \( 0 \leq x \leq 1 \). (Just set up the integral.)

\[
\int \int \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = \int \int (2x - 2y) dA
\]

\[
= \int_{0}^{1} \int_{x^3}^{x^2} (2x - 2y) dy \, dx
\]