Answer each of the following questions. Show all work, as partial credit may be given.

1. (5 pts. each) Consider the plane curve \( \mathbf{r}(t) = 2t \mathbf{i} + t^2 \mathbf{j} \).
   
   (a) Find the unit tangent vector \( \mathbf{T}(t) \) for the given curve.
   
   (b) Find the curvature, \( \kappa(t) \), of the given curve as a function of \( t \). (Hint: \( \kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} \).)

2. (10 pts. each) Find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) for each of the following functions.
   
   (a) \( f(x, y) = x^2 - xy^2 + 2y^3 \)
   
   (b) \( f(x, y) = \frac{x^2}{x^3 + y^3} \)
   
   (c) \( f(x, y) = e^{-x^2} \cos(x^2 - y) \)

3. (10 pts. each) Find \( f_{xx}, f_{yy}, f_{xy} \) and \( f_{yx} \) for each of the following functions.
   
   (a) \( f(x, y) = x^4 + 2x^2y + y^2 \)
   
   (b) \( f(x, y) = \sin(3xy) \)

4. (10 pts.) Find \( f_{xzx} \) for \( f(x, y, z) = e^{xyz} \).

5. (10 pts.) Suppose that the radius, \( r \), and height, \( h \), of a right circular cylinder are changing with time, \( t \). Find an expression for the rate of change of the volume, \( V \), of the cylinder as a function of \( r, h \), and their rates of change. (Hint: The volume \( V \) of a right circular cylinder of radius \( r \) and height \( h \) is \( V = \pi r^2 h \).)

6. (8 pts. each) Consider the function \( f(x, y, z) = xy + yz^2 + xz^3 \).
   
   (a) Find the gradient of \( f \).
   
   (b) Find the directional derivative of \( f \) at the point \((1, 0, 2)\) and in the direction \( \mathbf{v} = (2, -1, 2) \).
   
   (c) Find the maximum rate of change of \( f \) at the point \((1, 0, 2)\).