We work through a few problems which can be modeled by first order ODE’s.

Exercise (Mixing problem).

- At time 0 a tank has 100 gal. of water with 10 lbs. of salt dissolved.
- Water containing 1/4 lb/gal of salt enters at the rate of 3 gal/min.
- The instantaneously mixed mixture leaves the tank at the rate of 3 gal/min.
- Let $q(t)$ denote the number of lbs of salt in the tank at time $t$.

a) Write down the IVP satisfied by $q(t)$.
b) Identify the various techniques you know for solving this ODE.
c) Solve the IVP for $q(t)$ by selecting the easiest of the above techniques.
Exercise (Another mixing problem).

- At time 0 a tank has 200 gal. of water with 100 lbs. of salt dissolved.
- The tank has a capacity of 500 gal.
- Water containing 1 lb/gal of salt enters at the rate of 3 gal/min.
- The instantaneously mixed mixture leaves the tank at the rate of 2 gal/min.
- Let $q(t)$ denote the number of lbs of salt in the tank at time $t$.

a) Write down the IVP satisfied by $q(t)$.

b) In terms of solving the IVP, how does this problem differ from the previous one?

c) Solve the IVP for $q(t)$.

d) How many lbs of salt are in the tank at the moment of overflow?

e) If everything was the same, except the tank has an infinite capacity (instead of 500 gal), is there a limiting concentration of salt in the tank?
Exercise (Financial problem).

- Say you invest $S_0$ dollars at an annual interest rate $r\%$, compounded continuously.
- Let $S(t) :=$ balance in the account after $t$ years.

(a) Find and solve the IVP satisfied by $S(t)$

(b) Suppose instead you start with $0.00, and you deposit $K$ annually in the account (assume the deposit is made continuously). Find and solve the IVP for $S(t)$.

(c) In (b) above, if $K = 1000.00$ and $r = 5\%$, how much is the balance in 10 years?

(d) In (b) above, if $K = 1000.00$, we’d like to know how much $r$ must be so that the balance is $15,000.00$ in 10 years. Why is it obvious to you that there is such an $r$? Mathematically how can you prove there is such an $r$? Show how you would calculate this $r$. 