On intersections and composites of minimal ring extensions

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Abstract

The inclusion of (commutative) rings $A \subset B$, is called a *minimal extension* if there are no intermediate rings, i.e., no ring R such that $A \subset R \subset B$. Given minimal ring extensions $B \subset D$ and $C \subset D$, conditions are studied under which the extension $A := B \cap C \subseteq B$ (similarly $A \subseteq C$) is either minimal or a finite sequence of minimal extensions. Field-theoretic examples show that additional conditions are needed for such conclusions. Minimal extensions naturally break down into different types, and the assumptions on the type of say $B \subset D$ impacts on the extension $A \subset C$. We also examine for given minimal extensions $A \subset B$ and $A \subset C$, what conclusions can be drawn about the composite $D = BC$ (when it exists) as an extension of B or C.

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