

Generalized rings: examples and applications

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Abstract

When I was introduced to the concept of ring, it was in terms of a commutative (additive) group with an additional operation on the underlying set: a (multiplicative) semigroup that commutes with addition (or "distributes," as we said in olden days). But it could equally have been introduced as a multiplicative semigroup with additional operations that commute with each other and with multiplication. But of course additional operations other than an additive commutative group might be interesting or useful.

In his thesis, Nikolai Durov presents an elaborate theory of generalized rings from roughly this point of view, and applies it algebraically to complete the arithmetic and geometry of the rational integers, much as the points at infinity complete affine geometry to projective.

I will exhibit some these ideas with simple constructions in the real numbers, the rational numbers, and especially a real quadratic extension of the rationals.

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