Math 351, Probability
Homework no. 5

1. Suppose that the joint pmf of random variables X and Y is given by $p(1,1) = 0.5$, $p(1,2) = 0.1$, $p(1,3) = 0.1$, $p(2,1) = 0.2$, $p(2,2) = 0.1$, $p(x,y) = 0$ otherwise.
   (a) Find the marginal pmfs $p_X(x)$ and $p_Y(y)$.
   (b) Are X and Y independent? Why or why not?
   (c) Find the conditional pmf $p_{X|Y}(x|y)$.

2. A pair of fair dice, one red and one blue, are rolled until at least one of them shows a 1. Let X be the number of times the red die is rolled and let Y be the number of times the blue die is rolled.
   (a) Find the joint pmf of X and Y. (Hint: The two dice are rolled the same number of times.)
   (b) Find the marginal pmf of X.
   (c) Find $E(X + Y)$.

3. The joint probability density function of X and Y is given by $f(x,y) = 2$, for $0 < y < x < 1$.
   (a) Sketch the region on which $f(x,y) > 0$.
   (b) Verify that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dydx = 1$.
   (c) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.
   (d) Are X and Y independent? Why or why not?
   (e) Find the conditional density $f_{X|Y}(x|y)$.

4. The joint probability density function of X and Y is given by $f(x,y) = xy$, for $0 < x < 1$, $0 < y < 2$.
   (a) Find the marginal densities of X and Y.
   (b) Find $P\{X < Y\}$.
   (c) Find the density function of $Z = X + Y$.

5. X and Y are independent uniform $(0,1)$ random variables. Find the density of (a) $min(X,Y)$, (b) $max(X,Y)$.

6. The expected number of typographical errors on a page of a certain magazine is 1. What is the probability that an article of 5 pages contains (a) 0 or (b) 3 or more typographical errors? Explain your reasoning!

7. Suppose that X and Y are independent normal random variables, with $\mu_X = 5$, $\sigma_X^2 = 4$ and $\mu_Y = -1$, $\sigma_Y^2 = 9$.

   (a) Find $P\{X > Y\}$.

   (b) Find $P\{X + Y > 3\}$.

8. Suppose that X and Y are independent exponential random variables, with $\lambda_X = 2$ and $\lambda_Y = 2$. Find the density of $Z = X + Y$.

9. If X, Y, Z are independent exponential random variables, each with parameter $\lambda = 1$, find the probability that the largest of the three is greater than the sum of the other two.

10. If X, Y, Z are independent random variables that are uniformly distributed over $(0, 2)$, find the density of $W = X + Y + Z$ and graph this pdf.