

Math 351, Probability
Homework no. 4

1. The cumulative distribution function of a random variable X is given

$$\text{by } F_X(x) = \begin{cases} 0 & \text{if } x < -1, \\ \frac{x}{4} + \frac{1}{4} & \text{if } -1 \leq x \leq 0, \\ x^2 + \frac{1}{4} & \text{if } 0 \leq x \leq \frac{1}{2}, \\ x & \text{if } \frac{1}{2} \leq x \leq 1, \\ 1 & \text{if } x > 1. \end{cases}$$

- (a) Find the probability density function of X .
- (b) Find the expected value of X .
2. Suppose that X is a uniform random variable on the interval $[-1, 1]$
- (a) Find the probability density function of X .
- (b) Find the expected value and variance of X .
3. If X is a normal random variable with $\mu = 3$ and $\sigma^2 = 4$, compute
- (a) $P\{X \leq 1\}$
- (b) $P\{1 \leq X \leq 5\}$
- (c) $P\{X > 2\}$
- (d) $P\{X \geq 4\}$
4. In a certain large city, 40% of all the inhabitants are democrats. 100 residents of the city are polled. Find an approximation to the probability that at least 38 and no more than 42 of the residents polled are democrats.
5. A fair die is rolled 50 times. Find an approximation to the probability that exactly 33 of the numbers rolled will be no greater than 4.
6. A random variable X has the exponential distribution with parameter $\lambda = 2$.
- (a) Write down the pdf and the cdf of X ;
- (b) Find $E(X)$;
- (c) Find $P\{X > \frac{1}{2}\}$;
- (d) Find a number c so that $P\{X > c\} = 0.5$.

7. A customer at HardwareRUs buys a light bulb. The store has determined that the lifetimes of its light bulbs are exponentially distributed with parameter $\lambda = 0.5$ years.
- If the customer buys a light bulb and it is still working after 2 years, what is the probability that it will still be working after 4 years?
 - If the customer buys 3 light bulbs, what is the probability that they will all be working after 2 years?
8. Calamity Jane goes to the bank to make a withdrawal, and is equally likely to find 0 or 1 customers ahead of her. The service time of the customer ahead, if present, is exponentially distributed with parameter λ . What is the cdf of Jane's waiting time?
9. Suppose that X is a uniform random variable on $[-1, 1]$.
- Find the density function of the random variable X^2 .
 - Find the density function of the random variable X^3 .
10. Let X be a random variable with pdf $f(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ \frac{1}{2} & \text{if } 0 \leq x \leq 1, \\ \frac{1}{2x^2} & \text{if } 1 < x < \infty. \end{cases}$
- Find the pdf of the random variable $\frac{1}{X}$.
11. Let X be a random variable with pdf $f(x) = \frac{1}{\pi(1+x^2)}$ for $-\infty < x < \infty$.
- Verify that $\int_{-\infty}^{\infty} f(x)dx = 1$.
 - Show that $E(X)$ does not exist.