Math 351, Probability
Homework no. 3

1. The probability mass funtion of a random variable X is given by $p_X(x) = cx^2$, for $x = -2, -1, 0, 1, 2$, $p_X(x) = 0$ otherwise. Find c.

2. Two fair dice are rolled. Let X be the absolute of the difference of the two numbers that appear.

   (a) Find the probability mass function of X

   (b) Find the probability density function of X.

   (c) Find the expected value of X.

3. A fair die is rolled six times. Let X be the number of times that the number obtained is larger than 4.

   (a) Find the probability mass function of X

   (b) Find the expected value of X.

   (c) What kind of random variable is X?

4. Suppose that X is a random variable with $P(X = 1) = 0.2$, $P(X = 2) = 0.3$, and $P(X = 3) = 0.5$.

   (a) Find $E(X)$.

   (b) Find $\mathrm{Var}(X)$.

   (c) Find $E(sin(\frac{\pi}{X}))$.

5. Let X be a geometric random variable with probability $p = \frac{2}{3}$.

   (a) Find $E(3^X)$

   (b) Find $E(2^X)$.

6. Let X be a Poisson random variable with parameter $\lambda = 2$.

   (a) Find $P(X = i)$ for $i = 1, 2, 3, 4, 5, 6$, and compare this to the answer for 2a).

   (b) Find the expected value of X.

7. An urn contains 12 balls, of which 4 are white and 8 are black. You draw six balls from the urn, without replacement. Let X be the number of white balls among the six that you draw.

(a) Find the probability mass function of X.

(b) Find the expected value of X.

(c) What kind of random variable is X?

8. The four foreign language classes at a certain school contain, respectively, 25,22,24 and 29 students. It is assumed that no student is in more than 1 of these classes. One of the 100 foreign language students is selected at random. Let X be the number of students in this student's class. One of the four professors is selected at random. Let Y be the number of students in this professor's class. Compute $E(X)$ and $E(Y)$.

9. Suppose two evenly matched basketball teams play a best-of-seven series, i.e. the first team to win 4 games wins the series. Let X be the number of games the series lasts.

   (a) Find $E(X)$;

   (b) Find $\mathrm{Var}(X)$.

10. If $E(X) = 5$ and $\mathrm{Var}(X) = 1$, find

    (a) $E(5X + 2)$;

    (b) $\mathrm{Var}(5X + 2)$.

11. A car dealer purchases cars for 18,000 dollars and sells them for 20,000 dollars. However, he is not allowed to return unsold cars. If his monthly demand is a binomial random variable with $n = 10, p = \frac{1}{2}$, approximately how many cars should he purchase so as to maximize his expected profit?

12. Cars enter a parking lot at a rate of 10 per hour.

    (a) What is the probability that no car enters the parking lot during the next half hour?

    (b) What is the expected number of cars that will enter the parking lot during the next half hour?

    (c) Given that four cars enter in the next half hour, what is the probability that two of them enter in the first five minutes of the half hour?