

Math 351, Probability
Homework no. 2

1. Ninety students, including Joe and Jane, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Joe and Jane end up in the same class?
2. A fair coin is flipped twice.
 - (a) What is the probability that both flips are heads, given that at least one flip was heads?
 - (b) What is the probability that both flips are heads, given that the first flip was heads?
3. In a certain city, 45 percent of residents are Klingons, 25 percent are Romulans and 30 percent are Vulcans. In a recent election, ten percent of the Klingons voted, half of all the Romulans voted, and all of the Vulcans voted. A resident is picked at random. Given that this resident voted in the election, what is the probability that the resident is
 - (a) Klingon;
 - (b) Romulan?
 - (c) What fraction of the residents voted in the election?
4. Consider 3 urns. Urn A contains 2 white and 5 red balls; urn B contains 7 white and 3 red balls; and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white, given that exactly 2 white balls were selected?
5. Given events A, B, C of a sample space, prove that
 - (a) $P(AB) \geq P(A) + P(B) - 1$;
 - (b) $P(ABC) \geq P(A) + P(B) + P(C) - 2$.
6. Suppose that two fair dice are rolled.
 - (a) Find the probability that the two numbers obtained are different.
 - (b) Given that the two numbers are different, find the probability that the larger number is 5.

- (c) Given that the two numbers are different, find the probability that the sum is even.
 - (d) Given that the sum is even, find the probability that the two numbers are different.
7. We have two jars, each initially containing an equal number of balls. We perform four successive ball exchanges. In each exchange, we pick simultaneously and at random a ball from each jar and move it to the other jar. What is the probability that at the end of the four exchanges all the balls will be in the jar where they started?
 8. An urn contains five white and four black balls. Four balls are transferred to a second urn. A ball is then drawn from this urn, and it happens to be black. Find the probability of drawing a white ball from the remaining three.
 9. I have two coins in my pocket, one of which is fair and one of which has two tails. I pick one of these coins at random from my pocket.
 - (a) If I flip it once and it shows tails, what is the probability that it is the two-tailed coin?
 - (b) If I flip it three times and it shows tails every time, what is the probability that it is the two-tailed coin?
 - (c) If I flip it n times and it shows tails every time, what is the probability that it is the two-tailed coin? What happens to this probability as $n \rightarrow \infty$?
 10. A fair die is rolled 10 times.
 - (a) What is the probability that exactly 5 of the ten rolls result in numbers that are less than or equal to 2?
 - (b) What is the probability that at least five of the ten rolls result in numbers that are less than or equal to 2?
 11. A bag contains 40 red marbles and 20 green marbles. Ten marbles are chosen from the bag, without replacement.
 - (a) What is the probability that exactly 5 green marbles are selected?
 - (b) What is the probability that at least five green marbles are selected?
 - (c) Compare your answers to those of the previous problem.

12. In a certain class, each of the n students will independently show up with probability p_g if the weather is good and with probability p_b if the weather is bad. If the probability that there is bad weather tomorrow is q , what is the probability that at least k of the n students attend tomorrow's class?
13. Suppose Joe has \$75 and Sam has \$5. They repeatedly roll a die. Each time the number on the die is 2 or less, Sam pays Joe 1. Each time the number on the die is 3 or more, Joe pays Sam 1. The game ends when one player has all the money. That player is the winner. What is the probability that Joe is the winner?